Longevity swaps for Longevity Risk Management in Life Insurance Products

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Abstract

Purpose – The life insurance industry has been exposed to high levels of longevity risk born from the mismatch between realized mortality trends and anticipated forecast. Annuity providers are exposed to extended periods of annuity payments. There are no immediate instruments in the market to counter the risk directly. This paper aims to develop appropriate instruments for hedging longevity risk and providing an insight on how existing products can be tailor-made to effectively immunize portfolios consisting of life insurance using a cointegration vector error correction model with regime-switching (RS-VECM), which enables both short-term fluctuations, through the autoregressive structure [AR(1)] and long-run equilibria using a cointegration relationship. The authors also develop synthetic products that can be used to effectively hedge longevity risk faced by life insurance and annuity providers who actively hold portfolios of life insurance products. Models are derived using South African data. The authors also derive closed-form expressions for hedge ratios associated with synthetic products written on life insurance contracts as this will provide a natural way of immunizing the associated portfolios. The authors further show how to address the current liquidity challenges in the longevity market by devising longevity swaps and develop pricing and hedging algorithms for longevity-linked securities. The use of a cointergrating relationship improves the model fitting process, as all the VECMs and RS-VECMs yield greater criteria values than their vector autoregressive model (VAR) and regime-switching vector autoregressive model (RS-VAR) counterpart's, even though there are accruing parameters involved.

Design/methodology/approach – The market model adopted from Ngai and Sherris (2011) is a cointegration RS-VECM for this enables both short-term fluctuations, through the AR(1) and long-run equilibria using a cointegration relationship (Johansen, 1988, 1995a, 1995b), with a heteroskedasticity through the use of regime-switching. The RS-VECM is seen to have the best fit for Australian data under various model selection criteria by Sherris and Zhang (2009). Harris (1997) (Sajjad *et al.*, 2008) also fits a regime-switching VAR model using Australian (UK and US) data to four key macroeconomic variables (market stock indices), showing that regime-switching is a significant improvement over autoregressive conditional heteroscedasticity (ARCH) and generalised autoregressive conditional heteroscedasticity (GARCH) processes in the account for volatility, evidence similar to that of Sherris and Zhang (2009) in the case of Exponential Regressive Conditional Heteroscedasticity (ERCH). Ngai and Sherris (2011) and Sherris and Zhang (2009) also fit a VAR model to Australian data with simultaneous regime-switching across many economic and financial series.

JEL classification – G22, C50

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Findings – The authors develop a longevity swap using nighttime data instead of usual income measures as it yields statistically accurate results. The authors also develop longevity derivatives and annuities including variable annuities with guaranteed lifetime withdrawal benefit (GLWB) and inflation-indexed annuities. Improved market and mortality models are developed and estimated using South African data to model the underlying risks. Macroeconomic variables dependence is modeled using a cointegrating VECM as used in Ngai and Sherris (2011), which enables both short-run dependence and long-run equilibrium. Longevity swaps provide protection against longevity risk and benefit the most from hedging longevity risk. Longevity bonds are also effective as a hedging instrument in life annuities. The cost of hedging, as reflected in the price of longevity risk, has a statistically significant effect on the effectiveness of hedging options.

Research limitations/implications – This study relied on secondary data partly reported by independent institutions and the government, which may be biased because of smoothening, interpolation or extrapolation processes.

Practical implications – An examination of South Africa's mortality based on industry experience in comparison to population mortality would demand confirmation of the analysis in this paper based on Belgian data as well as other less developed economies. This study shows that to provide inflation-indexed life annuities, there is a need for an active market for hedging inflation in South Africa. This would demand the South African Government through the help of Actuarial Society of South Africa (ASSA) to issue inflation-indexed securities which will help annuities and insurance providers immunize their portfolios from longevity risk.

Social implications – In South Africa, there is an infant market for inflation hedging and no market for longevity swaps. The effect of not being able to hedge inflation is guaranteed, and longevity swaps in annuity products is revealed to be useful and significant, particularly using developing or emerging economies as a laboratory. This study has shown that government issuance or allowing issuance, of longevity swaps, can enable insurers to manage longevity risk. If the South African Government, through ASSA, is to develop a projected mortality reference index for South Africa, this would allow the development of mortality-linked securities and longevity swaps which ultimately maximize the social welfare of life assurance policy holders.

Originality/value – The paper proposes longevity swaps and static hedging because they are simple, less costly and practical with feasible applications to the South African market, an economy of over 50 million people. As the market for MLS develops further, dynamic hedging should become possible.

Keywords Longevity risk, Longevity swaps, RS-VECM, Variable annuities

Paper type Research paper

1. Introduction

Retirement schemes are increasingly dependent on individuals to collate personal savings for retirement through defined contribution accrual orders. At retirement, these accumulated funds will be used to finance an increasing and uncertain lifetime [1]. The traditional longevity risk products have been life annuities, indexed annuities and deferred annuities offered by life insurers. Annuity markets have not developed except in a few economies, typically where there has been an element of compulsion to annuitize retirement savings [2]. An innovative product introduced into South Africa and popular in Australia, USA, Japan and Europe is the variable annuity [3] with a guaranteed lifetime withdrawal benefit (GLWB) rider. The GLWB is crafted to cover insurance against longevity and market risks enabling individuals to have more flexibility and liquidity compared to life annuities (Ngugi *et al.*, 2015) [4]. In light of pandemics such as SARS and Coronavirus outbreak, (COVID-19), just to mention a few, annuity providers especially in developing economies incur enormous losses as a result of surges in mortality rate. As a hedging strategy against this exposure, Blake *et al.* (2006) propose longevity-linked securities whose cash-flows are linked to the longevity of an underlying population. However, interestingly, according to Crawford *et al.* (2008) longevity risk though is the largest risk exposure faced by today's annuity providers and life insurers threatening their existence, surprisingly it is still least understood. But, swap deals help insurers and annuities providers hedge against unexpected exposure like surges in claims by changing their risk profiles and taking the opposite direction of the underlying asset's risk exposure. Thus, swaps as a derivative instrument, can mitigate losses from longevity risk that retirement and insurance portfolios encounter not only in emerging and developed countries but also in developed economies (such as Italy, Spain, UK and US especially in light of global pandemics, which affects more the aged population) by matching assets and liabilities (cash flows and payouts).

The annuity market in South Africa, which makes it more interesting to study is that it is still developing whereas those extensively covered in the antecedent literature such as the UK, Australia and USA are relatively well developed and matured, with US annuity sales totaling $232.1bn in 2018 [5] and UK variable annuity sales spiking to $75.4bn in 2018 third quarter (LIMRA) [6]. Figures 1 and 2 reports South Africa and Belgian's annuity sales (both term and life annuities) with a growing trend. Variable annuities have proved to be lucrative, addressing consumer needs over issues such as bequests, early death and income flexibility, which are mostly encountered and more prone to emerging and developing economies, South Africa included, where low levels of expectancy are more prevalent. For over three decades, Ngai and Sherris (2011) observed that across the developed world where research and evidence are more concentrated, increases in survival probabilities led to the underestimation of longevity risk of individuals. In the case of emerging and developing economies, where survival probabilities are low due to high mortality rates and life expectancy within the range of 30–50 years, reliance on such evidence from developed countries when modeling longevity risk is futile.

A series of annuities have been issued in the South African and Belgian markets including fixed term, fixed life and inflation-indexed products. Figures 1 and 2 compare annuity sales done by gender in respect of South Africa and Belgium for life expectancy age of 65 years over the period 2014–2016. A more detailed comparison of life expectancies at birth between the two countries are reported in Appendix 1. Trends in Appendices 3 and 4,

Figure 1.
Life expectancy at 65, South Africa

Figure 2.
Life expectancy at 65, Belgium

which complement Figure 1, show life expectancies the annuity sales for South Africa over almost six decades (1960–2016) and more than one and half decade (2002–2017), respectively, whereas that for Belgium is presented in Appendix 2. Annuity sales have been up by about 11% in the third quarter of 2018 to $170bn. Most annuities sold in South Africa and Belgium are term certain annuities, with life annuities summing to about 7% and 11% of the total annuity sales in 2018 in South Africa and Belgium, respectively.

A viable life annuity market requires that life insurers be able to effectively control the risks of retail products without sitting on excessive capital stocks. Longevity risk can be mitigated using a number of methods including reinsurance and hedging in financial markets (Ngai and Sherris, 2011; Blake *et al.*, 2009). As reinsurance providers have been reluctant to shoulder substantial longevity risk amounts, insurance policy providers are increasingly contemplating hedging and securitization. Financial market hedging needs mortality (longevity) linked securities and products. Pooling longevity risk and natural hedging are not effective tools for methodical longevity management. This affects all age groups and can only be hedged in financial markets or by reinsurers through diversification with relatively uncorrelated risk profiles (Ngai and Sherris, 2011; Loeys *et al.*, 2007).

There have been inventions to shift longevity risk to the capital markets including mortality-linked securities (MLS), longevity swaps and longevity derivatives (like *q*-forwards). There is a potential role for government-issued longevity bonds to provide hedging (Blake *et al.*, 2009). At the same time, regulatory capital requirements for European insurers under Solvency II and risk-based capital requirements in other economies need to be allowed for in the successful provision of longevity insurance to individuals (Ngai and Sherris, 2011; Blake *et al.*, 2009). Despite the successful securitization of extreme mortality risk, there is not yet successful securitization of longevity risk (for example, Biffis and Blake, 2009; Lin and Cox, 2005) [7].

There is scant literature on annuities and longevity risk from an insurer's view in African markets setup (Ngai and Sherris, 2011). Bauer and Weber (2008) examine the risk of annuities for a single premium immediate life annuity using stochastic modeling to characterize the market mortality interactions. Ngai and Sherris (2011) investigate the usefulness of static hedging strategies for longevity risk management in Australia using a vector correction model with regime-switching (RS-VECM).

A vast amount of techniques available to hedge longevity risk focus on European or developed economies (for example, Blake *et al.*, 2014, UK; Ngai and Sherris, 2011, Australia; Roy, 2012, Asia; Yermo, 2000, Organisation for Economic Cooperation and Development countries). To the best of our knowledge, there exist no longevity swaps to counter risk in African markets. Moreover, it is almost impossible to dynamically hedge against longevity risk. Against this background, we develop longevity swaps and use existing hedging strategies for longevity risk management [8]. In this research, we develop market and mortality models to assess the effectiveness of longevity swaps and hedging instruments of mortality risk in the South African context. We assess a number of hedging strategies for portfolio annuities extending Ngai and Sherris (2011) and Dowd *et al.* (2006)'s studies. Our paper focuses on longevity risk, as there are well-developed markets for equity and interest rate risk in the developed economies. In South Africa, there is an infant market for inflation hedging and no market for longevity swaps. The effect of not being able to hedge inflation guarantees and longevity swaps in annuity products is revealed to be useful and significant particularly using developing or emerging economies as a laboratory.

This paper is organized as follows. Section 2 provides a survey of life annuities and hedging strategies, including longevity swaps, using MLS to mitigate longevity risk in South Africa and Belgium. We develop market models for market and longevity risk management in Section 3. The results of our analyzes are contained in Section 4. Section 5 concludes with a summary of the paper.

2. Hedging instruments

We consider a number of potential retail products suited for longevity risk for individuals, namely, longevity swap, inflation-indexed annuity, life annuity, deferred annuity, inflation-indexed deferred annuity and a variable annuity with GLWB. A range of policies is assessed consisting of 2,000 male policies of initial age 65 reflecting the current retirement age in South Africa and Belgium. This methodology is adopted from Ngai and Sherris (2011), Bauer and Weber (2008) and Dowd *et al.* (2006). The investigation focuses on a cohort of only ages 65 to design a simpler comparison. The terminal age presumed is ω = 108, so that the time frame for the simulations is T = 43. For longevity swaps, immediate and variable annuities, the initial payment is set to be R1,200,000.00 For deferred annuities a reduced early deposit of R1,12,000.00 is initiated, to enable a comparable level of longevity risk at the older age levels. For variable annuities, the initial lump-sum settlement is invested in a portfolio of a money market account and equity index, with the individual shouldering all the market risk. A GLWB rider to cover longevity risk is included. Each retail product is valued as accurately as practical replicating available data on market prices.

To examine longevity risk and effectiveness of static hedging strategies, the insurer's final surplus after all policyholders have died is simulated using market and longevity models calibrated to South African data. Claims, and regular GLWB charges for longevity swap and variable annuity, depend on mortality and market outcomes for inflation-indexed and variable annuities. Longevity swaps and static hedging strategies [9] are designed to match the expected liabilities. The insurer is assumed to have the ability to invest in risk-free bonds of varying maturities, a money market account and an equity portfolio. Following receiving the initial premium and overheads for the static hedge for the charges any remaining initial premium is invested in a T-year risk-free bond. In each subsequent year, the surplus is invested in a money market pool, which compounds at a risk-free rate. In this case, the insurer is assumed to borrow basing on this money market pool to counter unfavorable cash flows. Investment of surplus in the money market pool enables consistency. Surplus at the time t, U_t, is the accumulated net cash flows from premiums, claims outgoings and net cash flows from assets traded in the hedging strategy. This surplus gained at a hurdle rate of $R^U{}_t$, is expressed as follows:

$$U_t = U_{t-1}(1 + R^u t) \qquad P_t = C_t + H_t \tag{1}$$

where

U_t = insurer's excess in period t;
P_t = aggregate premiums/fees in period t;
C_t = aggregate claims outgoing in period t; and
H_t = net hedging cash flows in period t.

Costs are not considered to guarantee consistency in the products.

3. Methodology and data

3.1 Methods

The market model adopted from Ngai and Sherris (2011) is a cointegration vector error correction model with regime-switching (RS-VECM). This enables both short-term fluctuations, through the autoregressive structure [AR(1)] and long-run equilibria using a cointegration relationship (Johansen, 1995a, 1995b, 1988), with a heteroskedasticity through the use of regime-switching. The RS-VECM is seen to have the best fit for South African data under various model selection criteria. Harris (1997, 1999) fits a regime-switching VAR model using South African data to four key macroeconomic variables, showing that regime-switching is a significant improvement over autoregressive conditional heteroscedasticity (ARCH) and generalised autoregressive conditional heteroscedasticity (GARCH) processes in accounting for volatility (Chekenya, 2019; Chekenya and Dzingirai, 2020). Ngai and Sherris (2011) and Sherris and Zhang (2009) also fit a VAR model to South African data with simultaneous regime-switching across many economic and financial series.

A universal RS-VECM with a lag of p can be expressed as follows:

$$\Delta y_t = \mu + \sum_{i=1}^{p-1} A_i \, \Delta y_{t-1} + BCy_{t-p} + \lambda_t(\sigma_t) \tag{2}$$

where y_t is the d-dimensional vector of observations, $y_t = y_t - y_{t-1}$ is the first-order difference, μ is a mean vector, A is a d x d parameter matrix of coefficients, with B being d x r and C being r x d matrices of rank r denoting the cointegration (equilibrium) pattern between the variables and $\lambda_t(\sigma_t)$ is a vector of regime-dependent multivariate white noise error terms with covariances ($\sum_{\sigma t}$):

$$\lambda_t(\sigma_t) = \sim N_d(0, \Sigma_{\sigma t}) \tag{3}$$

Two regimes are used in the model, to proxy for a normal state and high-volatility state correspondingly as follows:

$$\sigma_t = \{0 \quad \text{normal} \quad \text{regime}\} \tag{4}$$

$$\sigma_t = \{1 \quad \text{high} - \text{volatility} \quad \text{regime}\} \tag{5}$$

The likelihoods of switching between regimes in a Markov chain with transition matrix P:

$$\mathrm{P} = \begin{pmatrix} p_0 & 1 - p_0 \\ 1 - p_1 & p_1 \end{pmatrix}. \tag{6}$$

where the likelihood p_0 and p_1 are constant over time.

The model is fitted with a two-stage procedure detailed in Krolzig (1997, 2013). First, to fit an RS-VECM we determine the cointegration rank r and matrix C using the Ngai and Sherris (2011) and Johansen (1995a, 1988) method. Second, the estimation procedure encompasses the use of an expectation-maximization (EM) algorithm to approximate the remaining parameters. The cointegration analysis and estimation are performed in *R* using the packages *urca* and *vars* developed by Pfaff (2008). The EM algorithm is performed using code written in *R,* which is checked by replicating the results in Ngai and Sherris (2011) and section 9.3 of Krolzig (1998), which includes an identical EM algorithm procedure [10].

We use lag of $p = 3$ based on model selection criteria, Corrected Akaike Information Criterion (CAIC) and Schwarz Information Criterion (SIC) and for parsimony [11]. The AICc is fitting for small samples, as it circumvents choosing models with excessive parameters and guarantees parsimony [12]. For a multivariate model of dimension d the model selection criteria are denoted as follows:

$$CIAC = \ln \mathrm{L} - \mathrm{k}\left(\llbracket n/(n-(j+1+d))\rrbracket\right) \tag{7}$$

$$SIC = lnL - k(\llbracket 1/2 \rrbracket \ln n) \tag{8}$$

where L is the log-likelihood, k is the number of parameters, n is the number of observations (time points), and j, is the number of structural parameters (per dimension). For comparison with the log-likelihood, we have defined the criteria so that a higher value is chosen and can be interpreted as a goodness of fit measure (log-likelihood) with a penalty for complexity.

The corresponding financial and economic series included in the model are defined as follows:

- $lnNights_t$ = Log Night Lights Data as a proxy for economic activity
- lnB_t = Log Bond Index [13]
- lnS_t = Log Stock Price Index [Johannesburg Stock Exchange (JSE) all share index]
- ln CPI_{t} = Log Consumer Price Index (CPI) to proxy Inflation
- Each market variable is an element of the time series vector y_t

The bank bill yield symbolizes the risk-free interest rate and is used to represent a short-term treasury bill yield due to data availability. The stock market index and interest rates are needed for the investments and capital stocks and inflation is essential for inflation-linked cash flows of the indexed annuities. On a new dimension, to circumvent the limitation, which among them, is the too strong assumption of homogeneity harvested best across citizens mostly plagued with gross domestic product, we measure economic activity using night lights data [14]. In the South African case, the use of night lights data to proxy economic activities is a motivated skewed state of development and high inequality levels across the breadth and citizens of the country. Unlike developed economies, where development is less skewed and inequality level is low, such high heterogeneity inequality and development level encountered in South Africa can be well captured by light intensity across the country.

3.2 Data

We collect market data from the South African Reserve Bank and JSE over the period 1990–2018. We used monthly time series data with 348 observations (19 years by 12 months). For stock prices, we used the JSE all shares monthly closing average for the index. We construct a South African luminosity monthly measure to proxy for economic activity as it yields better statistical results compared to GDP. For night lights data, we used night light satellite imagery provided by the Defense Meteorological Satellite Program, which covers the whole planet between latitudes 65° N and 65° S at the resolution of a 30 s arc, roughly equivalent to an area of 0.81 km^2 (less than 0.31 sq. miles) near the equator. We generated monthly stable night lights by averaging only the night lights during the dark half of lunar cycles and when days are shorter. This data for South Africa was then filtered to remove light due to forest fires and any other natural source such as northern and southern lights so that the result is an image at the country level of lights generated mostly by human activity. The luminosity or light intensity is a digital number between 0 and 63 where four zero refers to no light while the top coding of 63 stands for maximum light. The top coding of 63 is an artifact of limitations of satellite sensors and it does point to a problem in measuring growth over time, especially in the densely populated parts of South Africa. For CPI we use monthly indices as proxies of South Africa's month on month inflation rates. To obtain the bond index the 90-day bank-accepted-bill yield was accumulated up to the end of the month.

4. Results

In this section, we present a statistical summary of the data. Table 1 shows the sample statistics.

Unit root testing is performed and shows that all the series are of the order $I(1)$ except ln B_t, which is of the order $I(3)$, which might be a result of high volatility induced by high South African interest rate inflation oscillations. Nevertheless, a specified model anchored on the first difference of ln B_t for replicated settings is more reasonable and brooding of history. The model based on Δ ln B_t replicates a great number of negative returns because of the downward drift in interest rates over the period 1990–2018, leading to inverse future simulated returns.

The cointegration matrix C is estimated as follows:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & -0.251 & 0.253 \\ 0 & 1 & 0.387 & 1.745 \end{pmatrix} \tag{9}$$

The long-run equilibrium is expressed by:

$$\mathbf{C}y_t = 0 \tag{10}$$

Which can be re-arranged as follows:

$$lnG_t = 0.251lnS_t - 0.253lnCPI_t \tag{11}$$

$$lnB_t = -0.387lnS_t - 1.745lnCPI_t \tag{12}$$

Economic activity (G_t) is inelastic and positively correlated with stock market performance (S_t) which denotes the value of net assets of a firm on the equities market raises the dollar value of economic activity whereas inflation (CPI_t), which is also inelastic, reduces the dollar value of the economic activity. Positive correlation between economic activity and performance of equities market confirm Bandura and Dzingirai (2019)'s evidence of a

Statistic	Δ ln $Nights_t$	Δ lnB_t	Δ ln S_t	Δ ln CPI_t
Mean	0.0064	0.0231	0.0128	0.0125
Standard Deviation	0.0105	0.0104	0.1045	0.0112
Skewness	−0.2134	0.7189	−1.6792	0.8239
Excess Kurtosis	1.3221	−0.4981	5.3416	1.1389
1st Quartile	0.0001	0.0152	−0.0189	0.0040
Median	0.0078	0.0190	0.0618	0.113
3rd Quartile	0.0131	0.0341	0.08162	0.0201
Minimum	−0.0311	0.0018	−0.4928	−0.0034
Maximum	0.0289	0.0392	0.2176	0.0455

Note: The table depicts a summary of sample statistics for market log quarterly returns for the four macroeconomic variables

Table 1.
Sample statistics

positive effect of financial development on economic growth. Supporting the loanable funds theory and unlike the positive relationship between economic activity and stock market performance, results reported in the second cointegration equation shows bond index (B_t), which includes accrued nominal return, is inelastic (elastic), negatively and strongly correlated with equities $\{S_t$ [15], [inflation rate (CPI_t)]$\}$. The observations from results presented in equations 11 and 12 are corroborated by results reported in matrix A, where, for example, there is high AR dependence between $\Delta \ln S_t$ and $\Delta \ln B_{t-1}$ and $\Delta \ln CPI_{t-1}$; $\Delta \ln B_t$ and $\Delta \ln S_{t-1}$; $\Delta \ln S_t$ and $\Delta \ln CPI_{t-1}$; $\Delta \ln S_t$ and $\Delta \ln Light_{t-1}$; and $\Delta \ln B_t$ and $\Delta \ln Light_{t-1}$ with corresponding values of $A_{32} = -1.511$; $A_{24} = -0.835$; $A_{23} = -0.682$; $A_{31} = -0.642\, A_{31} = 0.521$; and $A_{21} = 0.415$, respectively. Furthermore, volatilities in $\sum_0$ are lower than those in $\sum_1$ an indication that regime 1 is a more volatile state than regime 0. Thus, South Africa's economic development in the previous period positively affects both debt and stock market but debt and stock market themselves are highly negatively correlate and inflation being a major factor, which reduces both debt and capital markets value and performance. This evidence indicates that South African life insurers and annuity providers must hedge longevity risk by adopting swap deals within the country, which involves debt market instruments and equities investment as a portfolio diversification strategy against idiosyncratic risk. As a hedging strategy against the systematic or country risk, which South Africa is currently faced with due to its downgrading on credit worthiness, longevity risk exposure can be reduced using international swap deals and reinsurance. Inflation more negatively affects the debt market (bond market) than it does on South Africa's economic activity.

The approximated parameters for the market model are as follows:

$$Щ = \begin{pmatrix} 0.172 \\ -0.022 \\ -0.381 \\ -0.288 \end{pmatrix}$$

$$A = \begin{pmatrix} -0.101 & -0.388 & 0.013 & 0.068 \\ 0.415 & 0.708 & -0.682 & -0.835 \\ 0.521 & -1.511 & 0.034 & -0.642 \\ 0.014 & -0.236 & -0.495 & 0.113 \end{pmatrix}$$

$$B = \begin{pmatrix} -0.015 & -0.021 \\ 0.003 & -0.007 \\ 0.057 & 0.158 \\ 0.034 & -0.013 \end{pmatrix}$$

$$\sum\nolimits_0 = 10^{-3} \begin{pmatrix} 0.0241 & 0.007 & 0.313 & -0.007 \\ 0.007 & 0.003 & -0.015 & 0.001 \\ 0.213 & -0.015 & 25.574 & -0.212 \\ -0.007 & 0.001 & -0.189 & 0.133 \end{pmatrix}$$

$$\sum\nolimits_1 = 10^{-3}\begin{pmatrix} 1.234 & -0.065 & 0.202 & 0.039 \\ -0.062 & 0.162 & -0.309 & 0.058 \\ 1.201 & -0.303 & 142.804 & -1.162 \\ 0.039 & 0.056 & -1.001 & 0.877 \end{pmatrix}$$

$$\mathbf{P} = \begin{pmatrix} 0.9 & 0.1 \\ 0.082 & 0.918 \end{pmatrix}.$$

The results show a strong AR dependency of ΔS_t on ΔG_{t-1} with:

$$A_{31} = 0.748 \tag{13}$$

Such that high economic activity (higher nighttime lights coefficient value) and productivity are linked to stock market development. The strong inverse correlation between ΔG_t and ΔB_{t-1} with:

$$A_{12} = -0.387 \tag{14}$$

This denotes an inconsistent pattern of rising interest rates. The strong dependence of S_t on shifts in the other macroeconomic variables as divergent from ΔS_t depicts that the stock market is strongly impacted by market factors in the short term.

The volatilities in Σ_1 are statistically significant and larger than those in Σ_0, with the error terms in Σ_1, being at least 4 times greater than those in Σ_0, depicting that regime 1 has a higher instability state.

To confirm the model selection criteria, and in the spirit of Ngai and Sherris (2011) and Gujarati (2009), in comparison to our more robust RS-VECM chosen model, we use a battery of alternative model specification techniques including vector autoregressive model (VAR), vector error correction model (VECM), regime-switching vector autoregressive model (RS-VAR) and the results are reported in Table 2.

Model comparison results in Table 2 indicate AR order denoted by ρ (AR(ρ), cointegration rank denoted by r and number of autoregression parameters denoted by k. On

Table 2.
Model selection criteria

Model Type	ρ	r	k	RS	ln L	CAIC	SIC
VAR	1	–	28	–	1,769.38	1,645.89	1,630.58
VAR	2	–	42	–	1,770.41	1,669.44	1,620.49
VECM	2	1	35	–	1,780.44	1,695.23	1,637.94
VECM	2	2	40	–	1,792.98	1,714.78	1,641.22
VECM	2	2	45	–	1,810.76	1,715.11	1,640.67
VECM	3	1	55		1,823.71	1,756.29	1,626.94
RS-VAR	1	–	40	Σ	1,828.11	1,778.77	1,731.73
RS-VAR	2	–	55	Σ	1,839.17	1,785.34	1,706.72
RS-VECM	2	1	57	Σ	1,851.20	1,791.89	1,739.66
RS-VECM	2	2	54	Σ	1,862.88	1,799.56	1,741.89
RS-VECM	2	3	50	Σ	1,905.38	1,810.69	1,743.78
RS-VECM	3	4	57	Γ, Σ	1,921.85	1,807.12	1,738.33
RS-VECM	2	2	58	Ξ, Σ	1,935.16	1,803.44	1,728.71
RS-VECM	2	1	100	A, Σ	1,948.78	1,790.97	1,710.56

the bases of model selection criteria, the results confirmed that the RS-VECM with a switching covariance matrix of 2 and a cointegration rank of 3 is the best model. In the case of AR orders greater than 3, we observed more complex results. For example, when AR orders increase from $\rho = 2$ to $\rho = 3$ and cointegrating rank from $r = 3$ to $r = 4$, a significant reduction in the criteria for the VECMs from their optimum orders of $\rho = 2$ and $r = 3$ was incurred. Also, the fitness of all the VECMs and RS-VECMs was found to be improved by the use of a cointegrating relationship, as regardless of additional parameters they obtained higher criteria values than their VAR and RS-VAR counterparts. Similarly, the use of regime-switching was beneficial, as the regime-switching models performed exceptionally better than their VAR and VECM counterparts. These results showed that RS-VECM is more robust and performs exceptionally well above all other veritable models tested in this study.

5. Conclusion

The study examines longevity risk and devises possible ways of counteracting the risk in South Africa. We develop a longevity swap using nighttime data instead of usual income measures as it yields statistically accurate results. We also develop longevity derivatives and annuities including variable annuities with GLWB and inflation-indexed annuities. Improved market and mortality models are developed and estimated using South African data to model the underlying risks. Macroeconomic variables dependence is modeled using a cointegrating VECM as used in Ngai and Sherris (2011), which enables both short-run dependence and long-run equilibrium. Regime switching is also used to model the idiosyncrasies of macroeconomic variables. Mortality is modeled with a multiple factor model and benchmark risk between the annuitant and population mortality is included.

Longevity swaps provide protection against longevity risk and benefit the most from hedging longevity risk. Longevity bonds are also effective as a hedging instrument in life annuities. The cost of hedging, as reflected in the price of longevity risk, has a statistically significant effect on the effectiveness of hedging options. There are currently no longevity swaps available in the South African (or Belgian) market. This study has shown that government issuance or allowing issuance, of longevity swaps, can enable insurers to manage longevity risk. If the South African Government, through the ASSA, is to develop a projected mortality reference index for South Africa, this would allow the development of MLS and longevity swaps. An examination of South Africa's mortality based on industry experience in comparison to population mortality would demand confirmation of the analysis in this paper based on South African data and other less developed economies. Our paper also shows that to provide inflation-indexed life annuities, there is a need for an active market for hedging inflation in South Africa. This would demand the government to issue inflation-indexed securities.

Furthermore, South Africa's economic development in the previous period was found to positively affect both debt and stock market but debt and stock market themselves are highly negatively correlated and inflation being a major factor, which reduces both debt and capital markets value and performance. Also, in support of the loanable funds theory, we observe debt markets represented by bond index and capital or equities market proxied by the stock market index to be negatively and strongly correlated. This evidence indicates that South African life insurers and annuity providers must hedge longevity risk by adopting swap deals with other players within the country, which involves debt market instrument providers and equities investments as a portfolio diversification strategy against idiosyncratic risk. As a hedging strategy against the systematic or country risk, which South Africa is currently faced with due to its downgrading on credit worthiness, longevity risk exposure can be reduced using international swap deals and reinsurance. Surprisingly, inflation more negatively affects the debt market (bond market) than it does on South Africa's economic activity. Thus, policymakers must thrive to reduce inflation, and concurrently, life insurers and annuities providers must hedge against inflationary risks by venturing in inflationary indexed swaps deals and correctly calibrate their stress testing models used for value at risk (VaR) modeling to correctly manage fat tails problem.

The paper proposes longevity swaps and static hedging because they are simple, less costly and practical with feasible applications to the South African market. As the market for MLS develops further, dynamic hedging should become possible. In light of pandemics like COVID-19, which asymmetrically affected economies and population demographics (in terms of mortality rates), as a suggestion for further studies, we recommend the application of the RS-VECM on comparative studies between developing, emerging and developed economies aiming at identifying best longevity risk management strategies and derivative products. Evidence from such comparative studies will provide answers to the following questions. Is longevity risk exposure dependent on country's stage of development? Does population demographics play any role in determining longevity risk exposure of a country? Which derivative instrument(s) that can best mitigate losses posed by longevity risk exposure and does such instrument(s) yield similar benefits to economies, which are at different developmental stages?

Notes

1. We gratefully acknowledge financial support from the Ghent University WOG research grant. We are also grateful for comments by Peter Bieter. We greatly acknowledge comments by two anonymous reviewers and the editor, which improved the paper. All, errors are ours alone.
2. For example, in South Africa, the introduction of tax-favored mandatory annuitization vehicles to which much of retirement saving has been directed.
3. Ngugi, A. M., Mare, E. and Kufakunesu, R. (2015). Pricing variable annuity guarantees in South Africa under a Variance-Gamma model. *South African Actuarial Journal*, *15*(1), 131–170.
4. The 2009 financial crisis has drawn attention to the risks for variable annuity writers resulting in insurers increasing fees, reducing benefits or withdrawing from selling variable annuities altogether (Burns, 2009; Lankford, 2010).
5. https://insurancenewsnet.com/innarticle/annuity-sales-spiked-14-in-2018-no-slowdown-in-sight#.XJNwPyIzb3g_for_a_commentary
6. https://www.investmentnews.com/article/20181126/FREE/181129967/variable-annuity-sales-up-25-from-this-time-a-year-ago
7. Lin and Cox (2005) propose a longevity bond with payments based on the insurer's loss experience eliminating basis risk while Wills and Sherris (2010) posit that age dependence is a critical element in the design and pricing of securitization.
8. We include longevity risk management products such as life annuities and variable annuities.
9. These strategies include fixed interest (zero coupon) bonds, longevity bonds and q-forwards.
10. Except with three regimes
11. Occam's razor.
12. For example, Ngai and Sherris (2011), Burnham and Anderson (2004).
13. Accumulated 90-day Bank-Accepted-Bill Yields.

14. Past studies have made extensive use of luminosity in socioeconomic studies. For example, a search on Google Scholar found almost 32900 studies since 1997 that have used luminosity data to proxy economic activity. Previous studies, primarily in the field of geoscience and economics, have used nighttime image data as a proxy for socioeconomic development of particular geographic areas (Doll *et al.*, 2000; Sutton and Costanza, 2002; Ebener *et al.*, 2005; Elvidge *et al.*, 2007, 1997; Sutton *et al.*, 2007; Henderson *et al.*, 2011, 2012). Elvidge *et al.* (2007, akon) conclude "nighttime lights provide a useful proxy for development and have great potential for recording humanity's presence on the earth's surface and for measuring important variables such as annual growth for development."
15. This denotes the value of real assets.

References

Bandura, W.N. and Dzingirai, C. (2019), "Financial development and economic growth in Sub-Saharan Africa: the role of institutions", *PSL Quarterly Review*, Vol. 72 No. 291, pp. 315-334.

Bauer, D. and Weber, F. (2008), "Assessing investment and longevity risks within immediate annuities", *Asia-Pacific Journal of Risk and Insurance*, Vol. 3 No. 1.

Biffis, E. and Blake, D.P. (2009), "Mortality-linked securities and derivatives", available at SSRN 1340409.

Blake, D. and Turner, J.A. (2014), "Longevity insurance annuities", *Benefits Quarterly*, Vol. 30 No. 1, pp. 39-47.

Blake, D., Cairns, A.J. and Dowd, K. (2006), "Living with mortality: longevity bonds and other mortality-linked securities", *British Actuarial Journal*, Vol. 12 No. 1, pp. 153-197.

Blake, D., Cairns, A., Dowd, K. and MacMinn, R. (2006), "Longevity bonds: financial engineering, valuation, and hedging", *Journal of Risk and Insurance*, Vol. 73 No. 4, pp. 647-672.

Blake, D.P. De Waegenaere, A. MacMinn, R.D. and Nijman, T. (2009), "Longevity risk and capital markets: the 2008-2009 update", available at SSRN 1362485.

Burnham, K.P. and Anderson, D.R. (2004), "Multimodel inference: understanding AIC and BIC in model selection", *Sociological Methods & Research*, Vol. 33 No. 2, pp. 261-304.

Chekenya, N.S. (2019), "Maximum likelihood estimation of stock volatility using jump-diffusion models", *Cogent Economics and Finance*, Vol. 7 No. 1, pp. 10-27.

Chekenya, N.S. and Dzingirai, C. (2020), "The impact of the presence of autoregressive conditional heteroscedasticity (ARCH) effects on spurious regressions", *Scientific African*, Vol. 8, p. e00382.

Crawford, T., de Haan, R. and Runchey, C. (2008), "Longevity risk quantification and management: a review of relevant literature", *Society of Actuaries*, Vol. 73.

Doll, C.N.H., Muller, J.P. and Elvidge, C.D. (2000), "Night-time imagery as a tool for global mapping of socioeconomic parameters and greenhouse gas emissions", *Journal of the Human Environment*, Vol. 29 No. 3, pp. 157-162.

Dowd, K., Blake, D., Cairns, A.J. and Dawson, P. (2006), "Survivor swaps", *Journal of Risk & Insurance*, Vol. 73 No. 1, pp. 1-17.

Ebener, S., Murray, C., Tandon, A. and Elvidge, C.C. (2005), "From wealth to health: modelling the distribution of income per capita at the sub-national level using night-time light imagery", *International Journal of Health Geographics*, Vol. 4 No. 1, p. 5.

Elvidge, C.D., Baugh, K.E., Kihn, E.A., Kroehl, H.W., Davis, E.R. and Davis, C.W. (1997), "Relation between satellite observed visible-near infrared emissions, population, economic activity and electric power consumption", *International Journal of Remote Sensing*, Vol. 18 No. 6, pp. 1373-1379.

Elvidge, C.D., Tuttle, B.T., Sutton, P.C., Baugh, K.E., Howard, A.T., Milesi, C., Bhaduri, B. and Nemani, R. (2007), "Global distribution and density of constructed impervious surfaces", *Sensors*, Vol. 7 No. 9, pp. 1962-1979.

Gujarati, D.N. (2009), *Basic Econometrics*, Tata McGraw-Hill Education.

Harris, G.R. (1997), "Regime switching vector autoregressions: a Bayesian Markov chain Monte Carlo approach", *Proceedings of the 7th International AFIR Colloquium*, Vol. 1, pp. 421-451.

Harris, G.R. (1999), "Markov chain Monte Carlo estimation of regime switching vector autoregressions", *Astin Bulletin*, Vol. 29 No. 1, pp. 47-79.

Henderson, J.V., Storeygard, A. and Weil, D.N. (2012), "Measuring economic growth from outer space", *American Economic Review*, Vol. 102 No. 2, pp. 994-1028.

Johansen, S. (1988), "Statistical analysis of cointegration vectors", *Journal of Economic Dynamics and Control*, Vol. 12, pp. 231-254.

Johansen, S. (1995a), "Identifying restrictions of linear equations with applications to simultaneous equations and cointegration", *Journal of Econometrics*, Vol. 69 No. 1, pp. 111-132.

Johansen, S. (1995b), *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*, Oxford University Press, New York, NY.

Krolzig, H.M. (1998), *Econometric Modelling of Markov-Switching Vector Autoregressions Using MSVAR for Ox*, Nuffield College, Oxford.

Krolzig, H.M. (2013), *Markov-Switching Vector Autoregressions: Modelling, Statistical Inference, and Application to Business Cycle Analysis*, Vol. 454, Springer Science and Business Media.

Lankford, A. (2010), "Do suicide terrorists exhibit clinically suicidal risk factors? A review of initial evidence and call for future research", *Aggression and Violent Behavior*, Vol. 15 No. 5, pp. 334-340.

Lin, Y. and Cox, S.H. (2005), "Securitization of mortality risks in life annuities", *Journal of Risk & Insurance*, Vol. 72 No. 2, pp. 227-252.

Loeys, J., Panigirtzoglou, N. and Ribeiro, R.M. (2007), *Longevity: A Market in the Making*, JPMorgan Global Market Strategy.

Ngai, A. and Sherris, M. (2011), "Longevity risk management for life and variable annuities: the effectiveness of static hedging using longevity bonds and derivatives", *Insurance: Mathematics and Economics*, Vol. 49 No. 1, pp. 100-114.

Ngugi, A.M., Mare, E. and Kufakunesu, R. (2015), "Pricing variable annuity guarantees in South Africa under a variance-gamma model", *South African Actuarial Journal*, Vol. 15 No. 1, pp. 131-170.

Pfaff, B. (2008), *Analysis of Integrated and Cointegrated Time Series with R*, Springer Science & Business Media.

Sajjad, R., Coakley, J. and Nankervis, J.C. (2008), "Markov-switching GARCH modelling of value-at-risk", *Studies in Nonlinear Dynamics & Econometrics*, Vol. 12 No. 3, pp. 1-29.

Sherris, M. and Zhang, B. (2009), "Economic scenario generation with regime-switching models", UNSW Australian School of Business Research Paper, (2009ACTL05).

Sutton, P.C. and Costanza, R. (2002), "Global estimates of market and non-market values derived from nighttime satellite imagery, land cover, and ecosystem service valuation", *Ecological Economics*, Vol. 41 No. 3, pp. 509-527.

Sutton, P.C., Elvidge, C.D. and Ghosh, T. (2007), "Estimation of gross domestic product at sub-national scales using nighttime satellite imagery", *International Journal of Ecological Economics and Statistics*, Vol. 8, pp. 5-21.

Wills, S. and Sherris, M. (2010), "Securitization, structuring and pricing of longevity risk", *Insurance: Mathematics and Economics*, Vol. 46 No. 1, pp. 173-185.

Further reading

Adelegan, O.J. (2009), "The derivatives market in South Africa: lessons for Sub-Saharan African countries", (No. 9-196), International Monetary Fund.

Barrientos, A. (2002), "Comparing pension schemes in Chile, Singapore, Brazil and South Africa", (No. 1651-2016-135989).

Barrientos, A. (2003), "Pensions and development in the South. The Geneva papers on risk and insurance", *Geneva Papers on Risk and Insurance – Issues and Practice*, Vol. 28 No. 4, pp. 696-711.

Barrieu, P., Bensusan, H., El Karoui, N., Hillairet, C., Loisel, S., Ravanelli, C. and Salhi, Y. (2012), "Understanding, modeling and managing longevity risk: key issues and main challenges", *Scandinavian Actuarial Journal*, Vol. 2012 No. 3, pp. 203-231.

Botha, E. and Makina, D. (2011), "Financial regulation and supervision: theory and practice in South Africa", *International Business and Economics Research Journal (IBER)*, Vol. 10 No. 11, pp. 27-36.

Deng, Y., Brockett, P.L. and MacMinn, R.D. (2012), "Longevity/mortality risk modeling and securities pricing", *Journal of Risk and Insurance*, Vol. 79 No. 3, pp. 697-721.

Donnelly, C. (2014), "Quantifying mortality risk in small defined-benefit pension schemes", *Scandinavian Actuarial Journal*, Vol. 2014 No. 1, pp. 41-57.

Dowd, K. (2003), "Survivor bonds: a comment on blake and burrows", *Journal of Risk & Insurance*, Vol. 70 No. 2, pp. 339-348.

Dowd, K., Blake, D. and Cairns, A.J. (2010), "Facing up to uncertain life expectancy: the longevity fan charts", *Demography*, Vol. 47 No. 1, pp. 67-78.

Frick, S.A., Rodríguez-Pose, A. and Wong, M.D. (2019), "Toward economically dynamic special economic zones in emerging countries", *Economic Geography*, Vol. 95 No. 1, pp. 30-64.

Hanewald, K., Piggott, J. and Sherris, M. (2013), "Individual post-retirement longevity risk management under systematic mortality risk", *Insurance: Mathematics and Economics*, Vol. 52 No. 1, pp. 87-97.

Kaseke, E. (2010), "The role of social security in South Africa", *International Social Work*, Vol. 53 No. 2, pp. 159-168.

Lee, R.D. and Carter, L.R. (1992), "Modeling and forecasting US mortality", *Journal of the American Statistical Association*, Vol. 87 No. 419, pp. 659-671.

Maurer, R., Mitchell, O.S., Rogalla, R. and Kartashov, V. (2013), "Lifecycle portfolio choice with systematic longevity risk and variable investment-linked deferred annuities", *Journal of Risk and Insurance*, Vol. 80 No. 3, pp. 649-676.

Meyricke, R. and Sherris, M. (2014), "Longevity risk, cost of capital and hedging for life insurers under solvency II", *Insurance: Mathematics and Economics*, Vol. 55, pp. 147-155.

Wang, J.L., Huang, H.C., Yang, S.S. and Tsai, J.T. (2010), "An optimal product mix for hedging longevity risk in life insurance companies: the immunization theory approach", *Journal of Risk and Insurance*, Vol. 77 No. 2, pp. 473-497.

Wong, T.W., Chiu, M.C. and Wong, H.Y. (2014), "Time-consistent mean-variance hedging of longevity risk: effect of cointegration", *Insurance: Mathematics and Economics*, Vol. 56, pp. 56-67.

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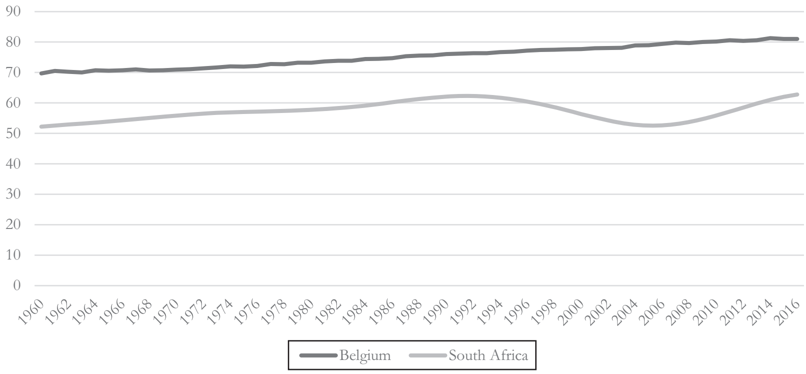
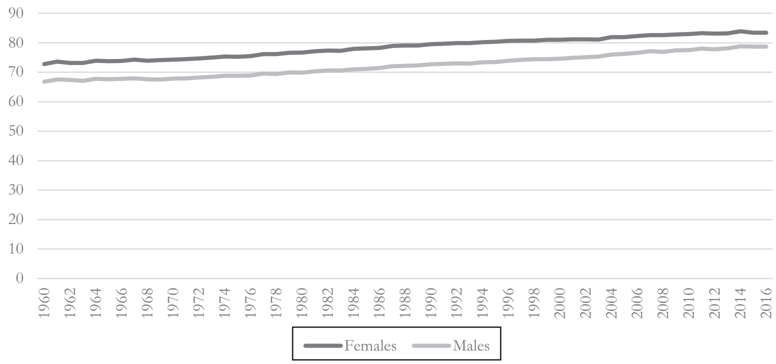
Appendix 1



Notes: The figure shows life expectancies at birth for South Africa and Belgium over the period 1960-2016. The figure show life expectancies at birth commonality between the two countries suggesting possible similarities in longevity risk exposure, thought that of South Africa has more oscillations

Figure A1.
Life expectancies at Birth (1960–2016)

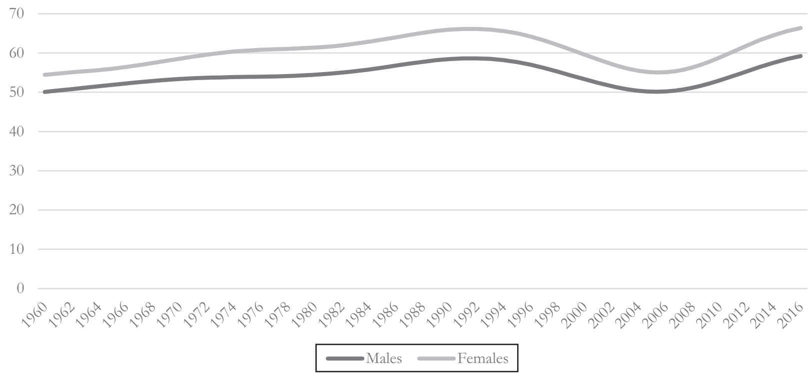
Appendix 2



Notes: This figure shows life expectancies for Belgium for the 1960-2016 period by gender. Thought with less oscillations than those for south Africa reported in Appendix 3, there is a rise in life expectancy rates over the period for both sexes

Figure A2.
Belgium's life expectancies by Gender (1960–2016)

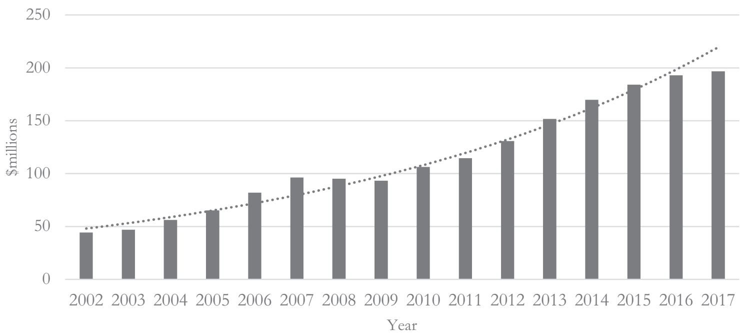
Appendix 3



Notes: This figure shows life expectancy for South Africa over the period 1960-2016 by gender. Thought with more oscillations than those witnessed in the case of Belgium reported in Appendix 2, there is a similar rising trend in life expectancy rates over the period for both sexes. This indicate similarities between the two countries

Figure A3.
South Africa's life expectancies by Gender (1960–2016)

Appendix 4



Notes: This figure shows total annuity sales for South Africa for the 2002-2017 period. There is a rise in sales over the period

Figure A4.
Annuity sales for South Africa (2002–2017)