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Modelling of a PZT Beam for Voltage Generation

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ABSTRACT

Piezoelectric materials, such as Lead Zirconate Titanate (PZT), have the ability to convert mechanical forces into an electric field in response to the application of mechanical stresses or vice versa. This novel property of the materials has found applications in sensor and actuator technologies, and recently, in the new field of energy harvesting. The study presented in this paper targets the modelling of a PZT beam device for voltage generation by transforming ambient vibrations into usable electrical energy. This device can potentially replace the battery that supplies power in microwatt range necessary for operating some wireless sensor devices. While the feasibility of this application has been repeatedly demonstrated in the literature, challenges for accurate modelling of mechanical to electrical energy generation in real world applications still remain. In this study, a simple mathematical model of simple voltage generation by a PZT beam device, based on classical beam analysis, was proposed. Experimental testing showed that the proposed model simulations are in good agreement with the experimental results. The energy harvesting capabilities of the fabricated PZT beam device were experimentally evaluated for possible application in low power wireless devices.

Keywords: Modelling, voltage generation, energy harvesting, piezoelectric material

INTRODUCTION

Energy harvesting is a form of renewable power generation and is a key enabling technology for a whole host of future distributed systems such as wireless sensors, implantable medical devices, and structural health monitoring. Among the possibilities for energy generators, piezoelectric materials have been generally used because of their ability to convert ambient mechanical energy into electrical energy (Chadrakasan *et al.*, 1988; Kim *et al.*, 2005a; Kim *et al*., 2005b). Meanwhile, recent advancements in the electronics industry have made it possible to reduce the power requirements of most electronic devices to levels that are comparable to the generation capabilities of piezoelectric harvester devices. Lead Zirconate titanate (PZT) is the most used piezoelectric material because of its high electromechanical coupling characteristics in single crystals (Sodano *et al.*, 2003; Sodano *et al*., 2004a,b). Analytical modelling is an inevitable element in the design process to understand various interrelated parameters and to optimize the key design parameters, while studying and implementing such power harvesting devices. Earlier work by Goldfarb & Jones (1999) presented a linear model of a PZT stack and analyzed its power harvesting capabilities. Their study showed that the maximum efficiency of the energy harvesting system occurs in a low frequency region which is much lower than the structural resonance of the PZT stack system. In addition, their

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work suggested that the efficiency of electrical power generation is related to the amplitude of the input forcing function due to the hysteresis behaviour of the PZT. Kasyap *et al*. (2002) developed a lumped element model to represent the dynamic behaviour of PZT in multiple energy domains using an equivalent circuit. They constructed flyback converter which allowed the impedance of the circuit to be matched to that of the PZT material so as to maximize power transfer from the PZT. Their model was experimentally verified using a one dimensional beam structure with peak power frequencies of around 20%. Roundy *et al*. (2003) studied the use of piezoelectric elements attached to vibrating beam to convert mechanical vibration energy for possible application in powering wireless sensor nodes. Their work demonstrated that when a piezoelectric energy harvester device was made to operate at frequency matching the vibrating frequency that is at resonance, the output electrical power generated was maximized. Meanwhile, Sodano *et al*. (2004a) developed an analytical model of a beam with attached PZT elements to provide an accurate estimate of the power generated by the piezoelectric effect. Their model was based on a more general model by Hagood *et al*. (1990); it incorporates the model by Crawley & Anderson (1990) in formulating the actuation equations for PZT device, and constitutive equations of bimorph actuators by Smit *et al.* (1991). The models by Crawley *et al*. and Smit *et al*. (1991) were developed for actuation. Later, Sodano *et al.* (2004) adopted these models for application in energy harvesting. In addition, the researchers also introduced an important addition by including 'material damping' which was excluded from the previous models. In this study, a theoretical model of voltage generation by a PZT element attached to a vibrating beam could be developed in the present study. Unlike the previous models which appeared to be mathematically complex, the approach used in this model is fairly simpler since it uses basic classical beam analysis and basic piezoelectric equations. The proposed model was experimentally investigated for validity.

THEORY AND SYSTEM MODELLING

Piezoelectric materials physically deform in the presence of an electric field, or conversely, produce an electrical charge when they are mechanically deformed. This effect is due to the spontaneous separation of charge within certain crystal structures, thereby producing an electric dipole. For a piezoelectric material, it is known that the output voltage of the material is a function of stress. Typically, stress is achieved through the displacement or bending of the piezoelectric beam.

Piezoelectric Materials

Piezoelectricity involves the interaction between electrical and mechanical behaviours of the material. Meanwhile, the static linear relations between the electrical and mechanical variables can approximate this interaction. In strain-charge form, these relations can be given by equations (1) and (2) (IEEE, 1988), as follows:

Strain –Charge form

$$D = \varepsilon^T E + dT \tag{1}$$

$$S = s^E T + dT \tag{2}$$

Where, T-Stress (N/m^2), S-Strain (m/m), E-Electric field Strength (V/m), and D-Dielectric displacement (C/m^2), s-Elastic compliance (m^2/N), ε-Permittivity (F/m), d-Piezoelectric constant (C/N or m/V).

In equations (1) and (2) above, the electrical quantities (E, D) have vector nature, while the mechanical quantities (T, S) have tensor nature of six components. In piezoelectric materials, the constants d, *e* and ε depend on the directions of electric field, displacement, as well as stress and strain. The piezoelectric strain constant (d) can be presented as d_{ij} which can be interpreted as charge created in the i^{th} direction under a stress applied in the j^{th} direction.

Based on piezoelectric cantilever, the most important piezoelectric strain coefficients when designing harvesting devices are d_{31} and d_{33} (Roundy *et al*., 2003). In the 3-1 mode, imposed strain in the 1-direction is perpendicular to the electric field in the piezoelectric material (i.e. in the 3-direction). In the 3-3 mode, the strain and electric field are parallel to each other (in the 3-direction) (IEEE, 1988)). Since the stress in a cantilever beam is always in the longitudinal direction, one can manipulate the poling direction within the material when configuring the electrodes on the piezoelectric material (*Figs. 1* and *2*).

The electrical–mechanical coupling of the 3-3 mode is higher than the 3-1 mode and the 3-3 mode piezoelectric cantilever will allow for a much higher open circuit voltage compared to a similarly sized 3-1 cantilever. Despite this limitation, 3-1 based cantilever energy harvesting systems are the most popular due to their simplicity and ease of integration into in a wide variety of structures and existing Micro-Electro-Mechanical Systems (MEMS). Another key advantage of the 3-1 mode of operation over the 3-3 system is that the 3-1 system is much more compliant, hence larger strains can be produced with smaller input forces. In addition, resonance frequency of the 3-1 systems is much lower than for the 3-3 mode (Sodano *et al*., 2003); therefore, the 3-1 mode is the mode of interest in this work.

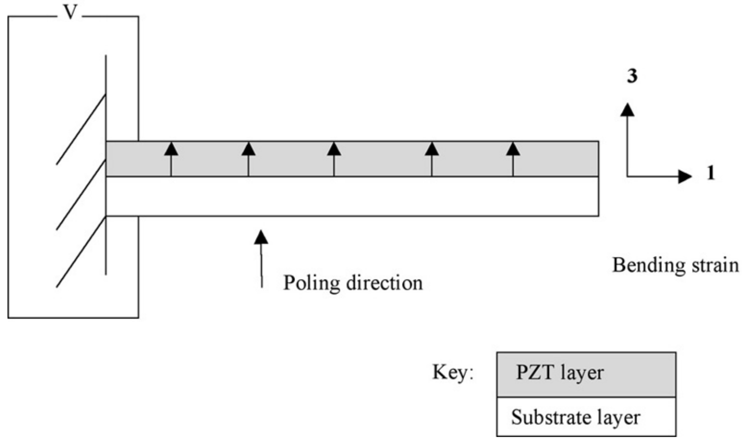


Fig. 1: A schematic view of a {3-1} mode harvester

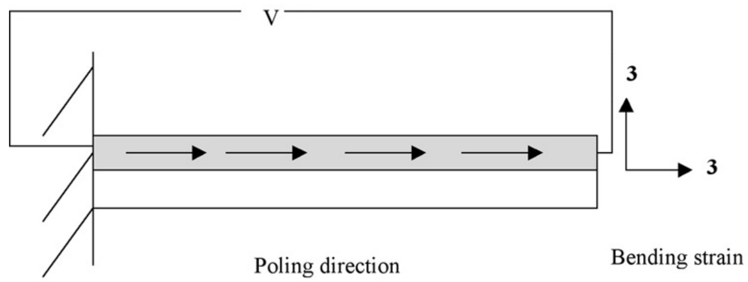


Fig. 2: A schematic of a {3-3} mode harvester

Model of PZT Voltage Generation

Electrical model of PZT as a voltage generator

The piezoelectric material can be modelled as an AC voltage source in series with a capacitor (*Cs*) and a resistor (*Rs*), as shown in *Fig. 3*.

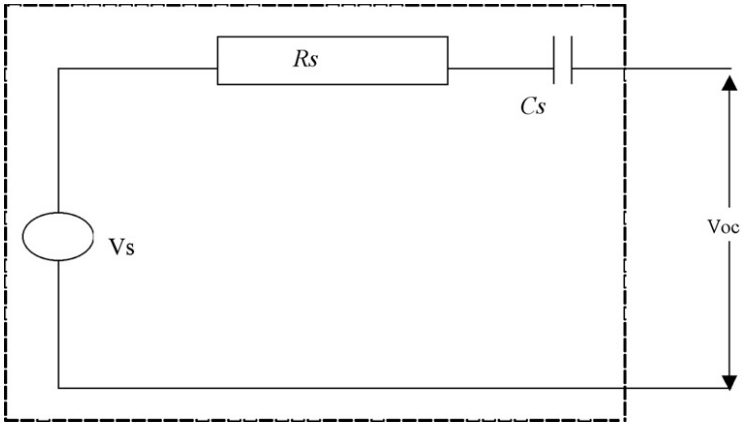


Fig. 3: PZT generator circuit model

The piezoelectric constitutive equation (1) can be rewritten as:

$$D_3(x,y) = \varepsilon_{33}^{T} E_3 + d_{31} T_1(x,y) \tag{3}$$

For an open circuit, the dielectric displacement (D) is zero. Thus

$$\varepsilon_{33}^{T} E_3 = -d_{31} T_1(x,y) \tag{4}$$

From the definition of uniform electric field strength:

$$E = \frac{V_{oc}}{t_p} \tag{5}$$

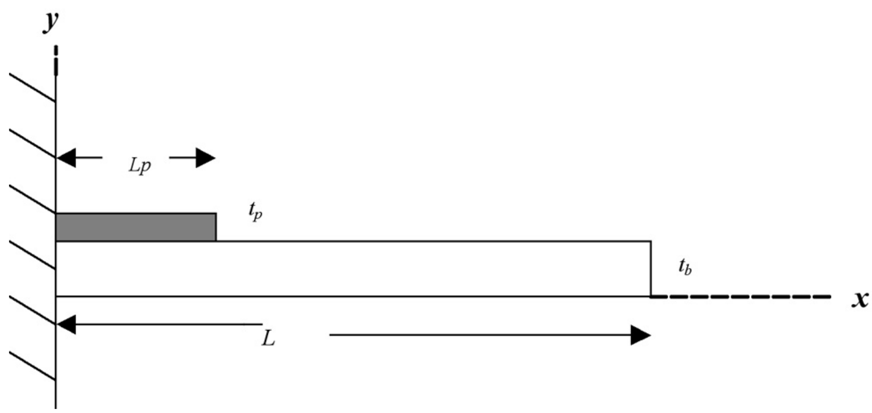
where t_p is the thickness of the piezoelectric material, and V_{oc} is the open circuit voltage Combining equations (4) and (5), the following will be yielded:

$$V_{oc} = -\frac{t_p d_{31}}{\varepsilon_{33}^T} T_1(x, y) \tag{6}$$

The next task is to use a simplified cantilever beam analysis to find an expression for the stress T_1 *(x, y)* at the surface of the PZT.

Cantilever beam analysis

The schematic view of the beam undertaken in this study is shown in *Fig. 4*. If the thickness of the host substrate layer *(t_b)* is far greater than the thickness of the PZT patch (t_p), and the beam length L is approximately ten times the length of the PZT layer (*Lp)*, the classical Euler-Bernoulli theory is then applied. Modelling is carried out for the case where the forcing function is a harmonic excitation.



t_b = thickness of substrate
t_p = thickness of PZT layer
Lp=length of PZT layer

Fig. 4: Cantilever beam made of piezoelectric patch and metal substrate

For a beam of uniform cross section, the beam curvature (k) is the second derivative of the beam deflection curve (Gere & Timoshenko, 1997):

$$\kappa = \frac{d^2 y}{dx^2} = \frac{M}{YI} \tag{7}$$

Where, M is the bending moment, Y the Young Modulus of the beam material and I is the Second Moment of Area about the principal axis.

Meanwhile, the average curvature is given by:

$$\bar{\kappa}(x) = \frac{1}{L_p}\int_0^{L_p} \frac{d^2 y}{dx^2} dx = \frac{1}{L_p} y'(L_p) \tag{8}$$

The average strain can be described in terms of the average curvature using the following relation:

$$S_1(x,y) = -\bar{\kappa}(y - y_c) \tag{9}$$

Where, y_c is the distance from the neutral axis of the PZT element.

The relation between the stress to the strain is $T_1(x,y) = E_p S_1(x,y)$. Using equations (8) and (9) in the relation, the following will be yielded:

$$T_1(x,y) = \frac{1}{L_p} E_p (y - y_c) y'(L_p) \tag{10}$$

Choosing the reference position to be the interface between the PZT and the substrate, the magnitude of stress at the surface of the PZT is then given by:

$$T_1(x,y) = \frac{1}{L_p} E_p t_p y'(L_p) \tag{11}$$

Assuming a static load at the end of the cantilever beam, the angular displacement (i.e. slope) of the tip of the beam (Urugal & Fenster, 1995) is as follows:

$$y'(x) = \frac{3a}{2L^3}(2Lx - x^2) \tag{12}$$

Where, $a = y(L)$ is the displacement at the cantilever beam end. Evaluating the slope at the end of the PZT element, $x = L_p$ and substituting in equation (11) gives:

$$T_1(x,y) = \frac{3a}{2L^3} E_p t_p (2L - L_p) \tag{13}$$

Substituting equation (13) into equation (6) gives the magnitude of the peak output voltage as:

$$Voc = \frac{3t_p^2 d_{31} a E_p}{2L^3 \varepsilon_{33}^T}(2L - L_p) \tag{14}$$

Assuming low vibrations and neglecting damping effects, the displacement at the tip end is the representative of amplitude of vibration of the tip of the beam. In this beam model, the forcing function is assumed to be sinusoidal. In the experimental setups, an electromagnetic shaker is used as the excitation source to drive the beam. The output voltage is essentially AC and has to be rectified and conditioned for it to be useful for applications such as powering sensors or battery charging. The power delivered to a particular load is a function of the ambient excitation frequency and the value of the load resistance relative to the impedance of the piezoelectric patch. The beam must be excited at its first resonance frequency to maximize the power output (Eggborn, 2003). Equation (14) describes a simple electromechanical model, where the output voltage is a linear function of amplitude of oscillation of the beam device.

MATERIALS AND METHODS

A thin PZT layer was attached to the aluminium substrate layer using two-part epoxy glue in a typical unimorph configuration (*Fig. 5*). The PZT model number PSI-5A4E, manufactured by Piezo Systems Inc., was used to cover the aluminium beam as the piezoelectric layer to 'fabricate the device'. This method has been successfully used in some previous studies by various researchers (*see* for instance, Chadrakasan *et al*., 1988; Sodano *et al*., 2003; Sodano *et al*., 2004a, b; Roundy

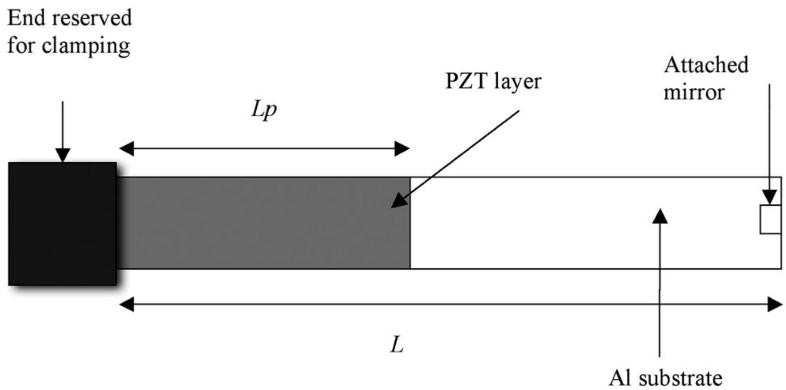


Fig. 5: The attachment of PZT on the aluminium beam

et al., 2003). The dimensions and properties of the materials are shown in Table 1. It should be noted that the aluminium substrate beam was made longer to ensure enough material was available for secure clamping. The actual clamped length of the aluminium beam and the PZT are 150 mm and 72.4 mm, respectively. The width of the PZT and aluminium beam was 25 mm.

TABLE 1
The dimensions and properties of PZT and aluminium beam

	Parameter	Value
PZT	length, L_p	72.4 mm
	width, w	25 mm
	thickness,t_p	0.267 mm
	Young modulus, Y_p	66 GPa
	Piezo-strain constant	190 E-12 C/N
	permitivity ε_{33}	320E-12 m/V
Al beam	length, L	150 mm
	width, w	25 mm
	Thickness	1.57 mm
	Young modulus, Y	70 GPa

Fig. 6 shows the experimental setup to test the device and to validate the proposed model. The device is clamped at one end and an electromagnetic shaker driven by a function generator is used to simulate the forcing function. When the forcing function was moved near the free end of the beam device, the deflections of the free end were observed to be too large to be accommodated by the shaker and its accompanying slender rod. This resulted in a distorted output voltage signal. In order to avoid this situation, an appropriate point of the location of the forcing function was

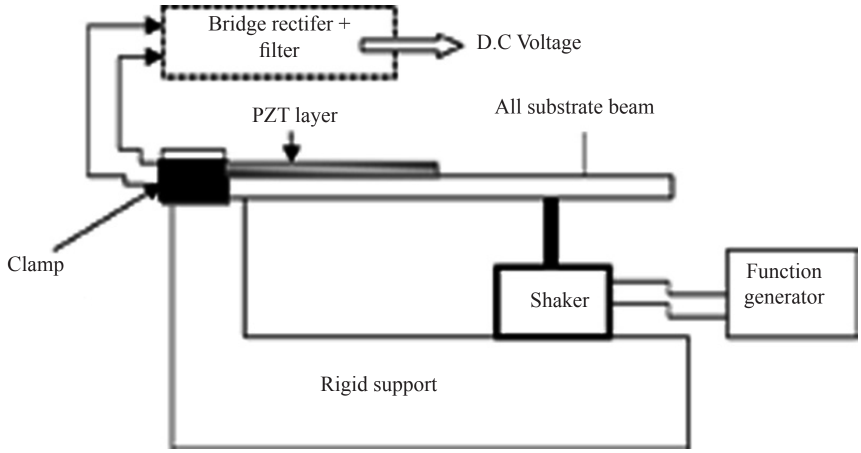


Fig. 6: The experimental setup

investigated and this was found to be 10.6cm from the clamped end of the device. It is crucial to highlight that this location was kept fixed throughout the experimental investigations. To measure the deflection amplitude at the tip of the beam, a reflective optical technique employing a He-Ne laser and a small mirror attached to the beam, was used (Putman *et al*., 1992).

The value of the first resonance frequency of the beam device was experimentally found to be 58.60Hz. With the function generator set at 58.60Hz, the open circuit voltage generated by the PZT was measured using an oscilloscope (HP 54610B) for different beam vibration amplitudes. The values were compared to the theoretical simulations generated from the suggested model. The results are discussed in the next section.

RESULTS AND DISCUSSION

The proposed model was simulated to investigate the relations represented by equation (14). The parameters used in the simulation study are the values indicated in Table 1, but with values of *Lp* ranging from 6 cm to 8 cm. Meanwhile, the value of *Lp* used in experimental investigation was 72.4mm. In this research work, only one length of the PZT layer was used in the experiments and it became the object of the ongoing study to use various lengths of PZT layer for further experimental validation of the model. The values of the open circuit voltage predicted by the suggested model are shown in *Fig. 7* below.

From *Fig. 7*, the output voltage is a linear function of amplitude of vibration. Varying the length of the piezoelectric layers affects the value of the output voltage, whereas the effect is dependent on the clamping conditions and the application point of the force inducing the vibrations. For a fixed value of *L*, the closer the PZT layer is to the point of stress application, the more the voltage output. The results of the open circuit voltage, predicted by the proposed theoretical model, were compared to the experimental results. Table 2 shows the values of the open circuit voltage as a function of the amplitude of vibration of the tip of the beam. A plot of the voltage versus amplitude of the vibration of beam tip for the experimental results and the theoretical model are shown in *Fig. 8*.

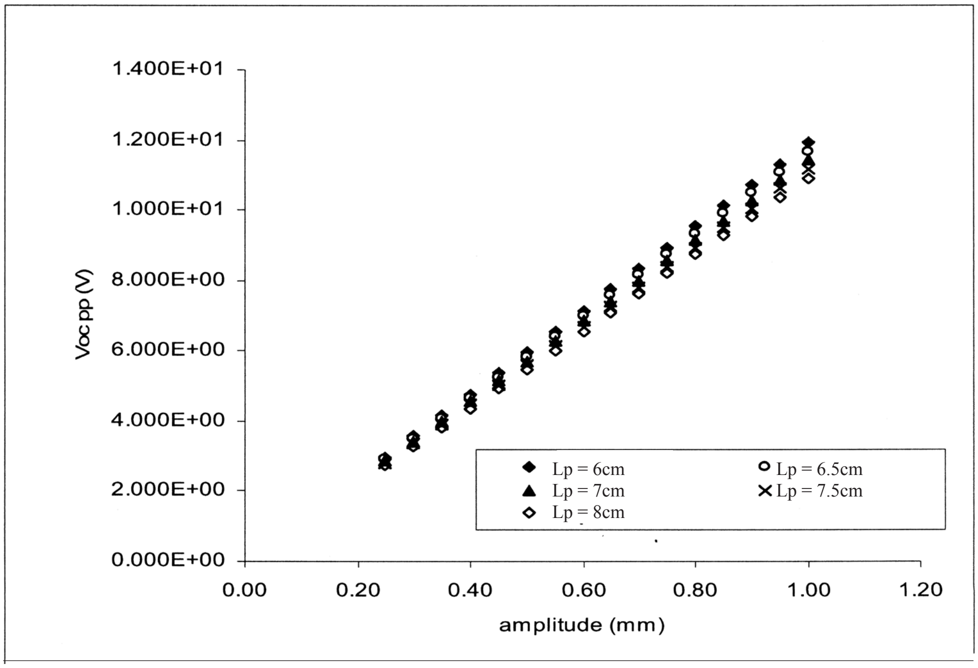


Fig. 7: Simulation results showing open circuit peak-to-peak voltage as a function of amplitude for various Lp values

TABLE 2
Measurements of amplitude and open circuit voltage

Amplitude of Al beam tip (mm)	Open circuit voltage-vocpp (V)		
	Experimental	Theoretical model	Absolute deviation
0.33	3.19	3.75	0.56
0.44	4.10	5.00	0.90
0.58	5.80	6.58	0.78
0.86	8.11	9.76	1.65
1.04	9.87	11.81	1.94
1.27	11.99	14.42	2.43
1.50	14.22	17.03	2.81
1.63	15.16	18.51	3.35
1.80	16.80	20.44	3.64

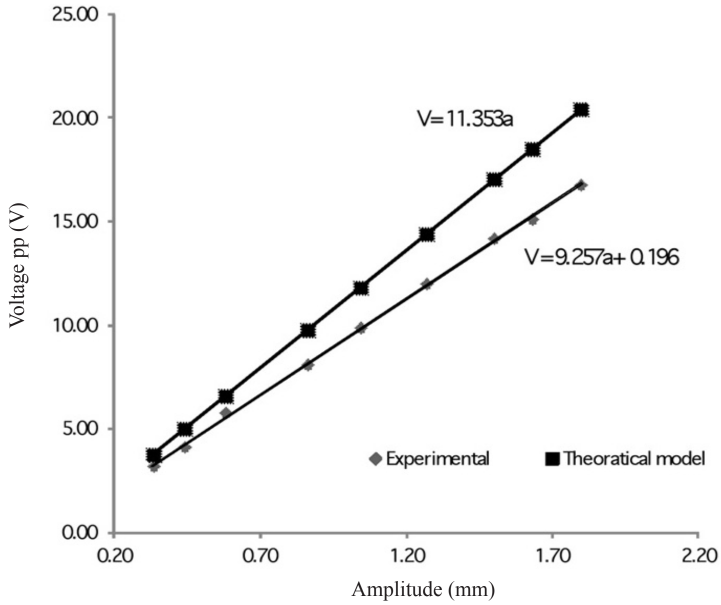


Fig. 8: The plot of the peak-to-peak output voltage as a function of amplitude

The theoretical model predicted a voltage output of 11.35 V per mm, while the experimental results suggested an output of 9.26 V per mm. The theoretically predicted voltage output is greater than the experimentally determined values by a margin of 2.09 V per mm. This observation was probably expected since the model did not take into account the effect of the glue-bonding layer. The glue bonding used had less elastic compliancy compared to the substrate beam; hence, not all stress was transmitted from the beam to the PZT element, resulting in reduced voltage generated by the PZT element.

ENERGY HARVESTING CAPABILITIES

Once the open circuit voltage was characterized and the theoretical model was validated, the next step was to investigate the power generation capabilities of the device. A full-wave bridge rectifier (*Fig. 9*) was constructed using Si diodes on a breadboard to convert the alternating voltage output from the harvester device into a steady DC signal.

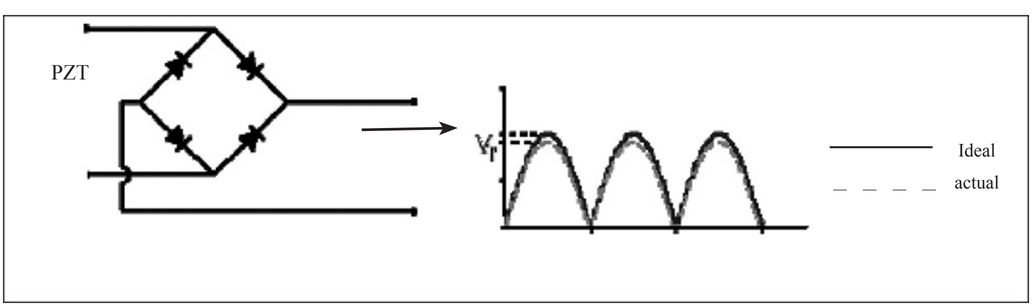


Fig. 9: Voltage output after signal is sent through a full bridge diode rectifier

After the preliminary investigation on the values of possible filter capacitors, a 3.3 μF capacitor was chosen because it was experimentally found to give the optimum DC output voltage (Nechibvute, 2008). It is important that the power measured was of a rectified - DC voltage signal, not the direct output from the device. This is a more accurate depiction of the useful power output of the harvester devices for low power electronics and wireless sensor applications. A load resistance, *R* was connected across the filter capacitor and the output voltage and the power output (*P*) calculated using the following equation:

$$P = \frac{V^2}{R} \tag{15}$$

where, *V* is the voltage delivered to the resistor of resistance *R*.

The values of R were varied and the corresponding values of voltage and power were also determined. These values were determined for the amplitude of the beam set at 1.69 mm and 1.19 mm. The power supply characteristic curves for the device are shown in *Fig. 10.*

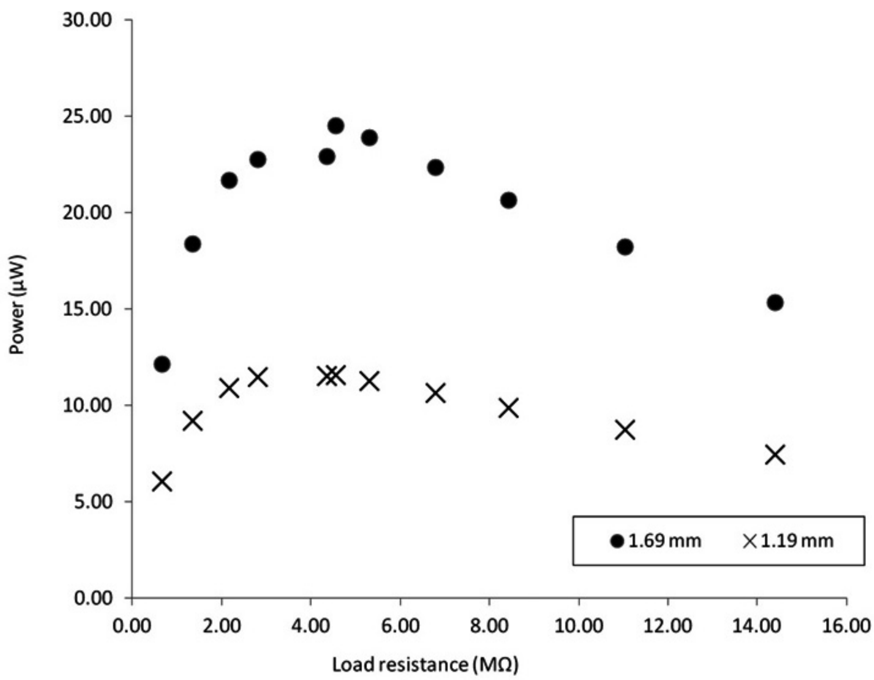


Fig. 10: Output power as a function of load resistance

From the power supply characteristic shown in *Fig. 10*, the optimum load resistance is about 4.6 MΩ. This value is very large and its magnitude is a fair representative of the output impedance of the piezoelectric harvester device. When the device is vibrating at the amplitude of 1.69mm, the maximum power it can deliver is 24.5 μW and the maximum current is 4.25 μA. At the amplitude of 1.19mm, the maximum power is 11.56 μW, and it can deliver up to 3 μA current. From the reviewed literature (Sazovoz, 2006; Carlos, 2007), the primary goal of vibration energy harvesting is to power wireless sensor devices. The power output of the studied beam system is relatively low compared to the real application requirements. As future research, the efficiency of the power harvesting circuitry needs to be optimized to allow a full amount of energy generated to be transferred to the load.

CONCLUSIONS

A simple but useful model of voltage generation by a piezoelectric harvesting beam device was successfully developed and experimentally validated. For the fabricated PZT beam, the proposed model predicted a voltage output of 11.35 V, while the experimental results suggested an output of 9.26 V, when the beam was vibrating at 1mm. This represented an agreement of up to 78% between the theoretical model and the experimental results. The power generation capabilities of the fabricated device were investigated for possible application in ultra low power applications. In future research, the model could be improved by incorporating the effects of damping and bonding layers on the voltage generated by the PZT. Furthermore, the intended location of the PZT on the host beam or structure needs to be studied so that its displacement can be optimized and the excitation range realized to allow for the tuning of power harvesting device and consequently increase the output power.

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