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A Lotka-Volterra competition model for modelling market competition in the telecommunication industry: Case Study of Zimbabwe.

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DEDICATION

I dedicate this dissertation to my Mum and Dad for their love and sacrifice.

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ABSTRACT

The purpose of this study is to use the Lotka-Volterra competition model to analyse the competition for market supremacy among the three Zimbabwean telecommunication companies namely, Econet, NetOne and Telecel. The parameters of the model are estimated using Genetic Algorithms. Firms' market shares have been used to reflect the competition among them. Estimation of market equilibriums and testing of the stability has been performed in this study and it has been shown that the long term outcome of the competition is the coexistence of the three companies. Our research results show that Econet will be the superior company in the long term. Future research can improve on the current methodology by including the promotional strategies being used by the companies in the model. The study can provide valuable information to the players for strategic planning and also for making informed decisions.

CHAPTER ONE

1.0 Introduction

The dynamics in the telecommunications industry market has since long back attracted the attention of many researchers with their major concern being to predict and forecast the future demand trends in the industry. In Zimbabwe, the act of liberalisation of the telecommunication industry and the further development of the country's infrastructure brought major changes in this industry. This process of liberalisation of the telecommunication industry enabled the entrance of other players in the sector, thereby introducing competition among them. The competition in the industry has intensified owing to promotions and value added services being introduced by the operators. In addition, the development in the telecommunications infrastructure has stimulated the fast development of other related services, for example e-commerce, internet banking and mobile banking. Furthermore, these developments have intensified the further adoption and penetration of mobile telephony, especially in rural areas. The introduction of value added services has contributed to an increase in the mobile penetration rate with many people purchasing SIM cards in order for them to enjoy the services. Before the liberalisation of the telecommunications sector in Zimbabwe, there was a monopoly in the telecommunications sector. The Postal and Telecommunications Act (Chapter 12:05), Act No 4 of 2000 came into action in 2000 to unbundle the Postal and Telecommunications Corporation (PTC) and enable the creation of the different entities to operate as successor companies with postal telecommunication and cellular licences. The primary aim of the liberalisation process was to achieve effective competition and promoting market structures and the process could enable the exploitation of substantial demand and innovation potentials in the telecommunication industry. The entrance of other players in the industry introduced a new environment for the market players and present new challenges in terms of both new opportunities and new

competitors. When liberalisation started, stiff competition emerged between the incumbent and the new companies that appeared, with all the companies aiming at increasing the number of their customers by providing services at attractive prices.

In Zimbabwe the liberalisation of the telecommunication sector enabled the entrance of companies like Econet and Telecel into the sector thereby beginning to compete with the already established NetOne which was the sole provider in that sector. The competition for subscribers between the three mobile network providers is intensifying continuously with the companies rolling out a series of promotions and in some cases, erecting billboards in a bid to outdo each other for customer attention and also in order to gain market share and retain or maintain their subscriber base. More so, the adoption of infrastructure sharing among the Zimbabwean mobile service providers enhanced further competition between them especially in the rural areas or marginalised markets.

The Zimbabwean telecommunications regulator, POTRAZ has reviewed that the mobile penetration rate has gone above the 100% mark to 103.5% in the 4th quarter of 2013. According to the report released by POTRAZ in the 4th quarter of 2014, NetOne has increase d its subscriber base with a massive 13.8% increase rate jumping from 2 379 285 to 2 707 682 active subscribers to become the second largest network provider. Telecel which used to be the second has been falling back as its active subscriber base has been dwindling, relegating it to 3rd position in terms of active subscribers. It used to command a sizeable number amounting to 2 400 729 in the 2nd Quarter report of 2014 and has dropped to 2 223 724 and recording the highest percentage drop rate of -7.37% meaning a total loss of 177 005 active subscribers. The loss of subscribers by Telecel was also witnessed by a decline in the Average Revenue Per User (ARPU) to US $4 in the 4th quarter of 2014 as compared to US $6 of the 3rd quarter of 2012. The fluctuation in the companies' shares shows that they are competitively interacting with each other with one company affecting the other negatively.

Currently POTRAZ reviewed in its report of the 1st quarter of 2015 that the total number of active mobile subscribers grew by 0.5% to reach 11.9 million from 11.8 million subscribers recorded in the previous quarter. The mobile penetration rate (active) also increased from 90. 3% recorded in the previous quarter to reach 90.8%. Active and total subscriptions per operator are shown in the table below:

Table 1.1 Active and Total Subscriptions per Operator

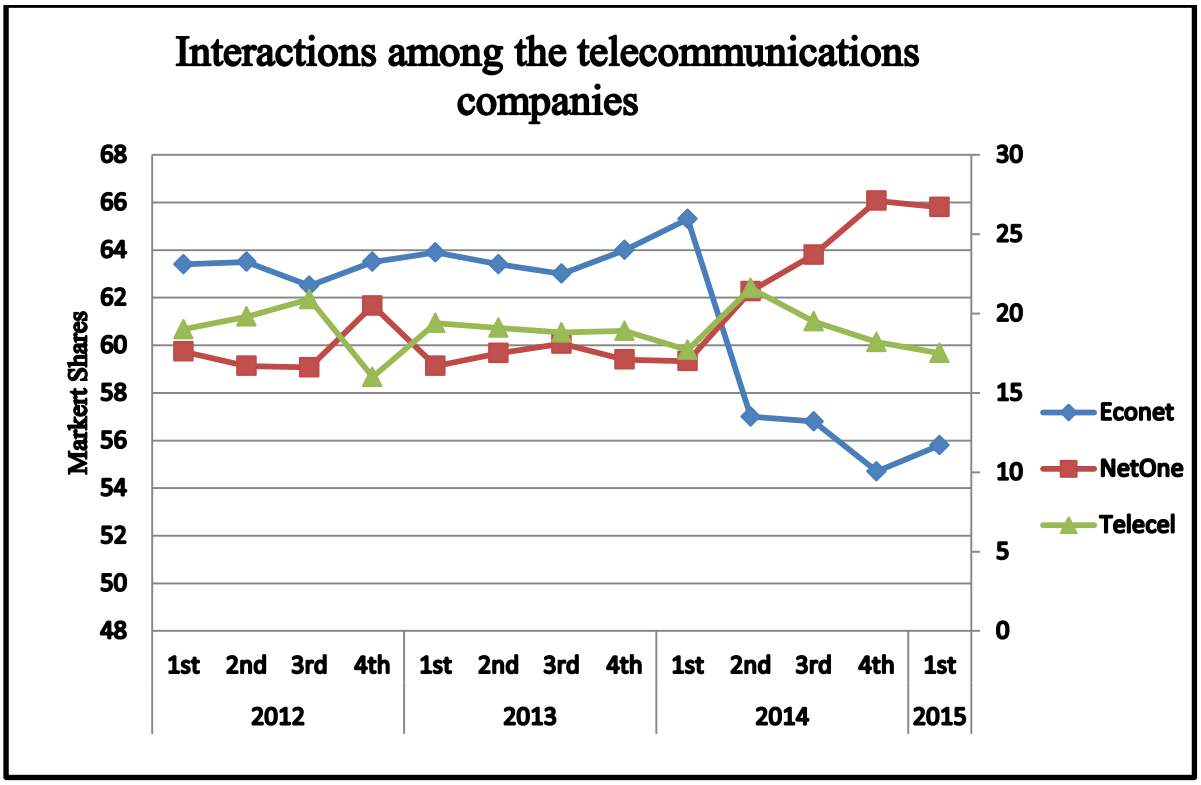
	Active Subscribers			**Total Subscribers**		
	4th Quarter 2014	**1st Quarter 2 015**	**% Change**	**4th Quarter 2014**	**1st Quarter 2 015**	**% Change**
Econet	6,451,863	6,619,058	2.6%	8,879,689	9,197,046	3.6%
Telecel	2,152,148	2,069,142	-3.9%	4,262,551	4,443,716	4.3%
NetOne	3,194,641	3,170,955	-0.7%	4,497,310	4,972,950	10.5%
Total	11,798,652	11,859,155	0.5%	17,639,559	18,613,712	5.5%

Source: POTRAZ, Operator Returns

Below is a graphical representation to show the interactions among the telecoms companies in Zimbabwe. It can be seen from the graph that as the shares of one company is increasing the shares of the other players tend to shrink. This shows that there is competition for subscribers among the players in the industry. In the first to the third quarter of 2012, Telecel's share was increasing and the effects were being felt among other players as witnessed by a shrink in the market share of NetOne and Econet though for the later the share wasn't significantly affected. More over an increase in the shares of Econet and NetOne resulted in a sharp decline in the share of Telecel between the 3rd to the 4th quarter of 2012. From the graph we can conclude that as one player's share increases, it gives a negative implication on the

growth of the other player(s). In the figure below, the scale for Econet is given at the left hand side and that of NetOne and Telecel is given at the right hand side. The choice of the secondary scale was due to the need to show the interactions among the shares of the service providers.

Fig 1.1 Interactions among the telecommunications companies from the 1st quarter of 2012 to the 1st quarter of 2015



This shows the need for the estimation of the evolution of market concentration such that the players will make informed decision basing on the estimates of their future shares. According to Baumol (1961) and Yadav (1995), the concept of market structure plays a central role to both economics and marketing. Both disciplines are concerned with strategic decision making . In decision-making analysis, market structure has an important role through its impact on the decision-making environment. The extent and characteristics of competition in the market

affect choice behaviour among the actors. Since market structure plays a pivotal role in the determination of market power, business behaviour and performance, studying the concentration of the new market is therefore necessary in order to identify its peculiarities, to describe competitor's behaviour and provide necessary inputs to the legislation and regulation authorities, regarding market structure and competition (Curry & George, 1983; Marfels 197 2) as well as predictions for the future regarding, among others, potential entry of new providers (Baye, 2006; Shy 1995). In addition the dynamics of market concentration plays a major role to the management of the telecommunication companies since it provides information that could assist in the decision making process concerning prospected competition from the rivalry firms. A volume of research has been carried out focusing on modelling and forecasting the concentration of markets and market competition between the market players in the telecommunication sector.

The present work focuses on the evolution of telephony market concentration in relation to the evolution of the corresponding market shares of the telecommunications companies or service providers in Zimbabwe and we use approaches from evolutionary theory of population biology and population dynamics.

According to Murray (2002) and Neal (2004), market evolution can be estimated and forecasted by making use of the Lotka-Volterra model. The model describes the interaction of species exhibiting or existing in a prey-predator mode. This is not the first time of these models being used to describe interactions however they have been mainly used to model market dynamics. Kim Lee and Ahn (2006) used them in the modelling of duopolistic market dynamics. Also Lee and Oh (2005) used the model to provide forecast for stock market. Market evolution has been estimated and forecasted in the previous years by making use of the Lotka-Volterra model. The model can also be used to explain or describe the interaction between two or more species that will be competing for a common but scarce resource.

1.1 Justification of the study

The telecommunication industry is dynamic, highly competitive and ever changing, thus the estimation and forecasting of market concentration and competition between the players involved becomes an imperative need.

1.2 Problem Statement

The three mobile telephone companies in Zimbabwe namely Econet Wireless Zimbabwe, Telecel Zimbabwe and NetOne have in the past years been the key actors in the mobile telephony war in Zimbabwe. The war was primarily for primacy in the small but lucrative market for telephony. This necessitates research for the estimation of the evolution of market concentration such that the players can make informed decisions basing on the estimates of their future shares. Furthermore market structure plays a pivotal role in the determination of market power, business behaviour and performance.

1.3 Aim

The aim of the study is to provide estimates of the degree of competition among the three Zimbabwe mobile telecommunication companies Econet, NetOne and Telecel and to be able to forecast market equilibrium and market concentration in the telecoms sector.

1.4 Objectives

- To use the proposed forecasting model in studying the dynamics of market structure and concentration making use of evolutionary theories from population biology.
- To provide an alternative way of estimating the level of concentration of markets in the telecommunications industry.
- To assess the performance of the proposed model by applying it previous data.

1.5 Significance of the study

The study will act as a source of information which can be used by the management of the telecommunication companies in making decisions or for strategy formulation. This study will act as a basis for the evaluation of the strategies and measure the effectiveness of the strategies.

1.6 Methodology: Overview and Assumptions

According to D. Neal (2004), population biology can be applied to many different areas and these include taxonomy, the study of the relationships of groups of organisms with each other and with their environment. More so, it can be used in the study of how the behaviours of different species are adopted from one generation to another. Population biology can also be used in theories which seek to study the interrelatedness of organisms from their previous generations.

Population dynamic refers to a branch of knowledge concerned with the sizes of populations and the factors that affect their maintenance, decline or expansion. It can also be seen as the study of the sequence of population changes characteristics of a particular organism. Freebase defined population dynamics as "a branch of life sciences that studies the short-term and long-term changes in the sizes and age composition of populations and the biological and environmental processes that influence those changes". Thus population dynamics deals with the way populations are affected by births and deaths rates and the immigration and emigration and it also studies topics such as ageing populations or population decline. This implies that the population modelling in this paper is an application of statistical models to study the changes in the populations and the interaction of the organisms with the environment, on their own (intra-species competition) and with other organisms of other species (inter-species competition). The major contribution of population modelling is its

ability to deduce if the competing species can coexist or they cannot coexist. More so it goes on to explore the factors affecting the coexistence of the competing species.

By making use of the above mentioned considerations, one can easily compare the relationship of the dynamics that describes competing species and the competition among the telecommunications companies towards gaining supremacy in the market with subscribers being the common scarce resource.

By assuming the properties of the model that describes the behaviour of species competing for a limited source, a methodology is developed to analyse the behaviour of mobile telephone service providers in Zimbabwe. The providers' market shares are used to estimate the interactions between them. More so, market shares are used as indicators of the level of concentration. It shows that firms are competing for a common source. Inter-species and intra-species competition shall also be put into consideration in order to predict the equilibrium solutions or points. Market shares can accurately indicate the degree of competition between the players since they can be considered as the observed outcomes of the non-cooperativeness of the firms with each firm seeking to maximise its own market share and thereby profits too. More so, market share can also be used to estimate the level of customers' switching among service providers.

1.7 Definition of key terms

Market structure: refers to the collection of factors that determines how buyers and sellers interact in the market, how prices change and how different levels of the production and selling process interact. It is divided into four types and these are oligopolies, monopolies, perfect competition and monophony.

Lotka-Volterra Model: These are equations that describe an ecological predator-prey or (parasite-host) model. They can be used to model the interaction or competition between species which will be competing for a common but limited resource.

Infrastructure sharing: It is the process in which telecommunication companies become partners in order to lower their increasing investments. The degree and method of sharing can vary from one country to the other depending on the regulatory and the competitive climates. It can also be identified as an operational strategy designed to minimise the resources need for communication infrastructure thereby making it much less costly and faster to deploy.

Intrinsic growth rate: the intrinsic growth rate of a population is defined as the maximal rate at which the population can grow under ideal conditions that is in the presence of unlimited resources, no competition, no predatation and no environmental stress.

Carrying Capacity: Population Matters (2011) defines it as the maximum number of individual that can be supported sustainably by a given environment without devastating or causing catastrophic effects to the environment.

Equilibrium Solutions: these are defined as constant solutions of a differential equation. They can also be known as critical points and they occur whenever the differential equation equals to zero.

Asymptotically Stable: this term is usually used when referring to a critical point say x_0 and it is said to be asymptotically stable if all trajectories which starts sufficiently close to x_0 will not only stay close to it but tends to approach x_0 as the time variable approaches infinity.

1.8 Abbreviations

PTC: Postal and Telecommunications Corporation

POTRAZ: Postal and Telecommunications Regulatory Authority.

1.9 Chapter Summary

Chapter 1 has presented the introduction of the research by describing the background of the telecommunications industry in Zimbabwe with a particular focus on the competition among the companies for market dominance. The problem statement, aim and significance of the study have also been explained in this chapter. Chapter focuses on some of the literature pertaining competing species.

CHAPTER 2:

LITERATURE REVIEW

2.0 Introduction

According to Central Queensland University library, literature review is an evaluative report of information found in the literature related to one's selected area of study. They go on to say that the literature review should describe, summarise, evaluate and clarify the literature. More so, they suggest that it should give a theoretical base for the research and it helps the author determine the nature of his/her research. Bell and Opie (2002) postulate that literature review requires sifting, selecting and ordering of material, including only important items and rejecting irrelevant or less significant items. Thus in this chapter, the work by a number of authors and scholars in different educational disciplines are to be used in providing an insight into the concept of market competition between firms or organisations. The definition of competing species, market competition and other important terms in the study are given in this chapter.

2.1 Market Competition

Many researches focusing on describing and modelling competitive forces and the impacts of promotional tools such as pricing and advertising have been carried out. However, competition has been met in literature. The major aim of including competition between players was to indicate that firms always behave non-cooperatively with each player seeking to maximise its own profits. In the past research papers, firms or organisations were assumed to be correctly anticipating their rival's strategy and how those strategies affects the other organisation's profits. In the context of competition, a number of models have been put forward, with the aim of forecasting market concentration. Diffusion models have been used in marketing as a tool for capturing the lifecycle dynamics of new products, forecasting the demand of new products and they also act as a decision aid in making pre-launch, launch and

post-launch strategic choices. Since the entrance of the diffusion models into marketing, they have become increasingly complex. The complexity has been due to the need to enhance the forecasting capability of these models and to improve their usefulness as a decision making tool for managers.

The Bass forecasting model is one of the diffusion models and it is an essential tool for forecasting the adoption of new products and new product categories. The Bass Model was first published in 1963 by Professor Frank M. Bass as a section of another paper. The section entitled "An Imitation Model" provided a brief, but complete, Mathematical derivation of the model from basic assumptions about market sizes and behaviour of innovators and imitators.

However, the paper did not provide empirical evidence in support of the model. Robinson and Lakhani (1975) further developed the model and they incorporated the effects of prices on the sales rate. Horsky and Simon (1983) further developed the Bass model by incorporating marketing variables coefficient of innovation as a function of advertising. It has its extended version, the generalised Bass Model (Bass, Krishnan and Jain 1994).

The generalised model expands on the original Bass model by including the effects of advertising and prices changes. Firms thus use the Bass forecasting model to develop marketing programs that estimate product sales rates for the future period on the basis of historical sales data of the product or comparisons of the product to the adoption rates of similar products. The Bass model of diffusion is used to estimate the speed new technologies will be adopted (Bass, Frank M, 1969). Mahajan and Muller (1979) have stated that the objective of diffusion models is to present the level of spread of an innovation among a given set of the prospective adopters over time. Michalakelis et al (2012) proposed a methodology for estimating the dynamics of the market exhibiting prey-predator behaviour. In their paper entitled "*Dynamic estimation of markets exhibiting a prey predator behaviour",* they

analysed the market dynamics of the telecommunications companies among five European countries.

In this paper however, the Lotka-Volterra model for competing species is used to model and forecast the competition between the telecommunications companies in Zimbabwe. The advantage of using the Lotka-Volterra model over diffusion models is that, the diffusion models requires transformation for them to include the effects of competition between the players or firms.

The use of the Lotka-Volterra competition model enables the estimation of market potential of each service provider and this can be used to formulate a system of equations that can be used to measure the effects of how each and every service provider affects the other. Lotka-Volterra models can also be used to measure numbers of individuals moving out of a collective group over a specific time frame and this also provides vital information regarding competition. In addition Genetic Algorithms are to be used in this paper in the process of estimating the parameter values.

2.2 Competing Species

Connell and Joseph (1961) defined competition between species as a negative interaction that occurs whenever two or more organisms require the same limited resources. In general, organisms require resources to grow, reproduce and survive for example animals require food in the form of other organisms and water, whereas plants require soil nutrients, like nitrogen, water and light. Organisms, however, cannot acquire a resource when other organisms consume or defend that resource. Therefore, competitors reduce each other's growth, reproduction or survival.

The rate of change of population can be hypothesised as being proportional to the current size of the population. A logistic model to describe the population growth of a species in the

absence of any competition was described by Boyce and DiPrima (1997) and D. Neal (2004). The following model has been proposed for such species:

$$\frac{dx(t)}{dt} = rx(t)\left[1 - \frac{x(t)}{K}\right] \qquad (1)$$

where (i) $x(t)$ is the population at time t , (ii) r is the growth rate and K is the saturation level or the environmental carrying capacity.

Some models basing on the above mentioned approach are widely used in modern literature for demand estimation and forecasting and these includes the logistic family growth models which were developed by R. Bewley & D. G Fiebeg (2005) and the Gompertz model that was put forward by I. P Rai (1999).However, in the case where two or more species coexist, there will be competition among them for the limited or scarce resources. According to D. Neal (2004) competition occurs when two or more individuals or species experience depressed fitness (reduced growth rates or saturation levels) attributable to their mutual presence in that area. Competition is most typically considered as the interaction of individuals that vie for a common resource that is in limited supply, but more generally can be defined as the direct or indirect interaction of organisms that leads to a change in fitness when the organisms share the same resource. The outcome usually has negative effects on the weaker competitors (Holomuzki *et. al* 2010). This approach therefore implies that, whenever two species or more each require a resource that is in limited supply such that the availability of the resource to one species tends to be negatively affected by the presence of the other species. The effects of the mutual presence will be reduced growth rates for each of them. (Agrawal *et al.* 2007; Ricklefs 2008; Brooker *et al*. 2009) argues that at the coarsest level, ecological interactions can be defined as either intra-specific or inter-specific. Intra-specific interactions are those that occur between individuals of the same species, while interactions that occur between two

or more species are called inter-specific interactions. However, since most species occur within ecological communities, these interactions can be affected by, and indirectly influence, other species and their interactions such that we may have competition and predation. If the growth rate of one population is decreased and the other increased the populations are said to be in a predator–prey situation and if the growth rate of each population is decreased then it is competition and also if each population's growth rate is enhanced then is called mutualism or symbiosis.

Since the telecommunications companies in Zimbabwe exist in a competitive market with each player seeking to maximise its market share and profits, the interaction between them may be best described by competition in which the growth rate of each population is decreased by the existence of the other.

A Lotka-Volterra model can be used to cater for the reduction of the growth rates of each species. The Lotka-Volterra model is a well-known model and it was named after its developers Lotka and Volterra. This model has been widely used to describe interactions among species. T. H Fay & J.C Greef (1985) gives a theoretical analysis and application of the interaction among three or more species. Thus basing on those analysis, the dynamics of the competition among the Zimbabwean telecommunication companies can be modelled in a similar way by considering them as n competing species. A system of non-linear differential equations can be used to represent the dynamics.

$$\frac{dx_i}{dt} = x_i\left[a_i - \sum a_{ij}x_j\right]; \quad i = 1,2,3\ldots,n \qquad (2)$$

Where (i) $\frac{dx_i}{dt}$ is the rate of change of the population of species i, (ii) a_i refers to the intrinsic growth rate of populationx_i , (iii) a_{ij} is a measure of the interspecies the strength of the

competition, a measure of the effects of one species on the growth of the other that is for $i \neq j$ and also a measure of intraspecies when $i = j$.

From (1), transformations can be done such that the system can be represented as below.

$$\frac{dx(t)}{dt} = rx(t)\left[1 - \frac{x(t)}{K}\right]$$

$$= x(t)\left[r - \frac{r}{K}x(t)\right]$$

$$= x(t)[r - ax(t)] \qquad (3)$$

Unlike in (2), the addition of terms that can capture the effects of one species on the growth of others can be easily incorporated in the above model (3)

2.3 Chapter Summary

Chapter 2 has presented the literature concerning market competition and competing species also diffusions of markets was explained in the chapter. A number of scholars who have applied the Lotka-Volterra model have also been discussed.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 Model Formulation

Citing the need to develop a better understanding of the competitiveness and dynamics of markets in the telecommunication industry, the use of a Lotka-Volterra competition model can make valuable contributions. Christopher J.L, Murray et al, (1996) as was cited in a research paper by Masarira Tinashe (2011), he cited that the major aim of any modelling exercise is to extract as much information as possible from the available data in order to provide an accurate representation of the knowledge and uncertainty about, in the case they were considering was epidemics. However, in this paper it shall be to gain a better understanding in the dynamics of telecommunications markets for the service providers in Zimbabwe. In the formulation of the model, the main assumptions has been the consideration of the corresponding market share values of the telecommunications companies, and consider them as n species that are competing for a common limited resource. The service providers are the competitors for the present and future customers of their service. We assume that there are only three service providers in Zimbabwe and these are, Econet, Telecel and NetOne. Other factors that might affect the dynamics of the markets of the telecommunications companies are considered constant during the period of consideration.

With the above stated assumptions in mind, systems of equations to describe the dynamics can be given by a generalised non linear differential equation below:

$$\frac{dx_i}{dt} = x_i\left[a_i - \sum a_{ij}x_j\right], \qquad i = 1,2 \dots m\ ,\ j = 1, 2, \dots m \qquad (4)$$

$where\ (i)\ x_i$, refers to the market shares of provider $i, (ii)\ \ a_{ij}\ i \neq j$, is a parameter that captures the influence of the interaction between the subscribers of different providers and $(iii)\ \ a_{ij}, i = j$, captures the effects of the subscribers of the same provider. This implies that the equations will be written as below for each and every player.

Let $x_1, x_2\ and\ \ x_3$denote Econet wireless, NetOne and Telecel Zimbabwe respectively. Thus the system of equations can be written as below.

$$\frac{dx_1}{dt} = x_1[a_1 - b_1x_1 - c_1x_2 - d_1x_3]$$

$$\frac{dx_2}{dt} = x_2[a_2 - b_2x_2 - c_2x_1 - d_2x_3] \qquad (5)$$

$$\frac{dx_3}{dt} = x_3[a_3 - b_3x_3 - c_3x_1 - d_3x_2]$$

Where, (i) $x_1, x_2\ and\ x_3$are the corresponding market shares of each of the players, (ii) $a_1,\ a_2,\ a_3, b_1,\ b_2,\ b_3,\ c_1,\ c_2,\ c_3,\ d_1, d_2$ and d_3 are positive constants. The model shows the presence of intra-population and inter-population interactions among the players.

In the absence of interactions among the players, their market share grows exponentially. The intra and inter-populations interaction inhibits the growths of the three players' shares. $a_1,$ $a_2$and $a_3$are the intrinsic market share growth rates, $b_1,\ b_2$and $b_3$are self-inhibitory effects within the three players and $c_1,\ c_2,\ c_3,\ d_1, d_2$and $d_3$represents the inhibitory between the three players that is the interspecies interference between the players.

According to R. Fildes & V. Kumar (2002), interspecies interaction measures the rate at which the subscribers to a service that discontinue their subscription to that service in a given time period.

3.2 Parameter estimation

The estimation of the parameters of the system of equations given in (2) was done using genetic algorithms. This in turn gives a base of the applicability of the Lotka-Volterra competition model in the telecommunication industry. In many previous researches, various methods were applied in the parameter estimation process. Among other methods used in parameter estimation were the Least squares method, the Maximum likelihood method and Judgemental method. However, in this paper, heuristic methods are employed by the means of genetic algorithms. The algorithms are applied over a particular data set in order to train the system.

3.3 Genetic algorithms

Genetic algorithms (GA) were introduced by Holland (Goldberg 1998; Holland 1975). They are adaptive heuristic search algorithms based on the evolutionary ideas of natural selection and genetics. As such they represent an intelligent exploitation of a random search used to solve optimisation problems. Though they are randomised, GA are by no means random, instead they exploit historical information to direct search into the region of better performance within the search space. The basic techniques of the GAs are designed to simulate process in natural systems necessarily for evolution, specifically those evolutions that follow the principles first laid down by Charles Darwin of the “Survival of the fittest”. The main argument being that, in nature, competition among individuals for scanty resources results in the fittest individual dominating over the weaker. The key points to a process are reproduction, crossover and mutation. Reproduction involves the copying or reproducing solution vectors, crossover involves swapping of partial solution vectors and mutation is the process of randomly changing a cell into the string of the solution vector thus preventing the possibility of the algorithm being trapped.

The algorithm is run until it reaches the optimal solution to the fitness function which is used to evaluate the individuals. As stated above, there are many ways which parameters can be estimated. However these approaches to estimation have some demerits which can be addressed by genetic algorithms and these include bias, multicolinearity, inefficient thus resulting in parameters with low forecasting power. As for the management judgement, it is associated with low level of reliability and high levels of bias.

Advantages of Genetic Algorithms

- Can estimate parameter values even with a minimum number of data points..
- They can address weaknesses associated with other techniques by effectively reducing standard errors for the parameters.
- They use parallel, evolutionary search algorithms to locate parameters that optimises the objective function (minimum sum of squared errors)

Draw backs of Genetic algorithms

- Requires specific software and this makes it impossible to perform them without the software.

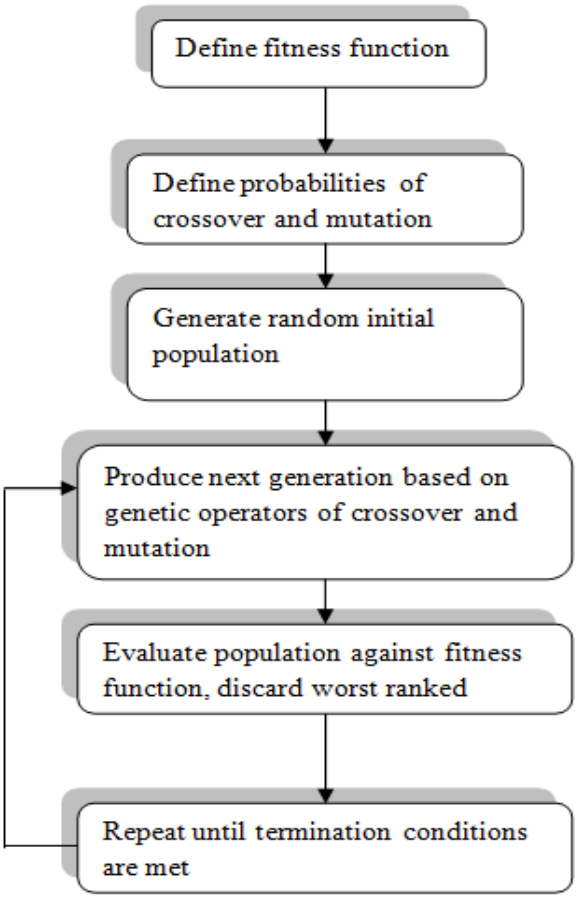
3.4 Steps in Genetic Algorithm parameter estimation

1. Define the fitness function for the particular optimisation problem.
2. Set crossover and mutation probabilities.
3. Randomly generate an initial population $N(0)$.
4. Generate $N(t+1)$ population by using probabilities to select individuals from $N(t)$ to produce off springs via genetic operators of crossover and mutation.
5. Compute fitness for each individual in the current population $N(t)$. Off springs with values that are closer to the fitness function are more probable to contribute with one

or more offspring to the next generation. Discarding off springs that diverge from the fitness function.

6. Repeat steps 4 and 5 until either a specified number of generations are created or after some predefined time has elapsed.

Procedure for the estimation of proposed system



The algorithm in this paper is performed for the system described by (2) with the following characteristics.

i. The objective function was formulated in such a way that it seeks to minimise the Mean Squared Error (MSE) between the observed values and the estimated values for each service provider's market share.

$$MSE = \frac{1}{T}\sum_{t=1}^{T}(x(t) - \hat{x}(t))^2$$

Where $x(t)$ and $\hat{x}(t)$ are the observed and the estimated values respectively for competitor i.

ii. Basing on the changes in the market shares, initial values of the parameters were estimated. In addition, the algorithm was executed with random initial values in order to ensure that the algorithm would converge to the global minimum, instead of being trapped to a local one.

iii. Stopping conditions, the algorithm is terminated when the reduction value becomes less than 0.01% in the last 10.000 iterations

iv. The population sizes were set to 500 individuals per generation.

3.5. Data sources

The data that has been used in this research has been obtained from POTRAZ and it can be accessed easily by visiting the official POTRAZ website. Thus secondary data has been empl oyed to achieve the objectives of the study. MATLAB and Microsoft Excel were the s oftwares used in data analysis.

3.5.1 Secondary data

Secondary data refers to the information that has been collected for a purpose other than the c urrent research project but has some relevance and utility in the research. As was cited in Dangaiso's research paper, he cited that according to Shiu et al (2009), secondary data refers to historical data structures of variables previously collected and assembled for some research problem or opportunity situation other than the current situation. He goes on to say that secon dary data is a viable alternative source of information which provides comparative and contex tual information that can result in other discoveries.

3.5.2 External Sources

Published Sources are those prepared for public distribution and are normally found in libraries or provided by variety of other entities such as trade associations, regulatory bodies, professional organisations or companies. POTRAZ (Post and Telecommunication Regulatory Agency of Zimbabwe) provide the statistics pertaining to the market shares and other related statistics in relation to competition in the telecommunication industry in Zimbabwe.

3.6 Chapter Summary

Chapter 3 presented the research methodology. The methods of analysis and the assumptions that were used in the research were presented in this chapter. Data sources have also been included in this chapter. The next chapter will dwell on the presentation of data, its analysis a nd discussion of findings.

CHAPTER 4

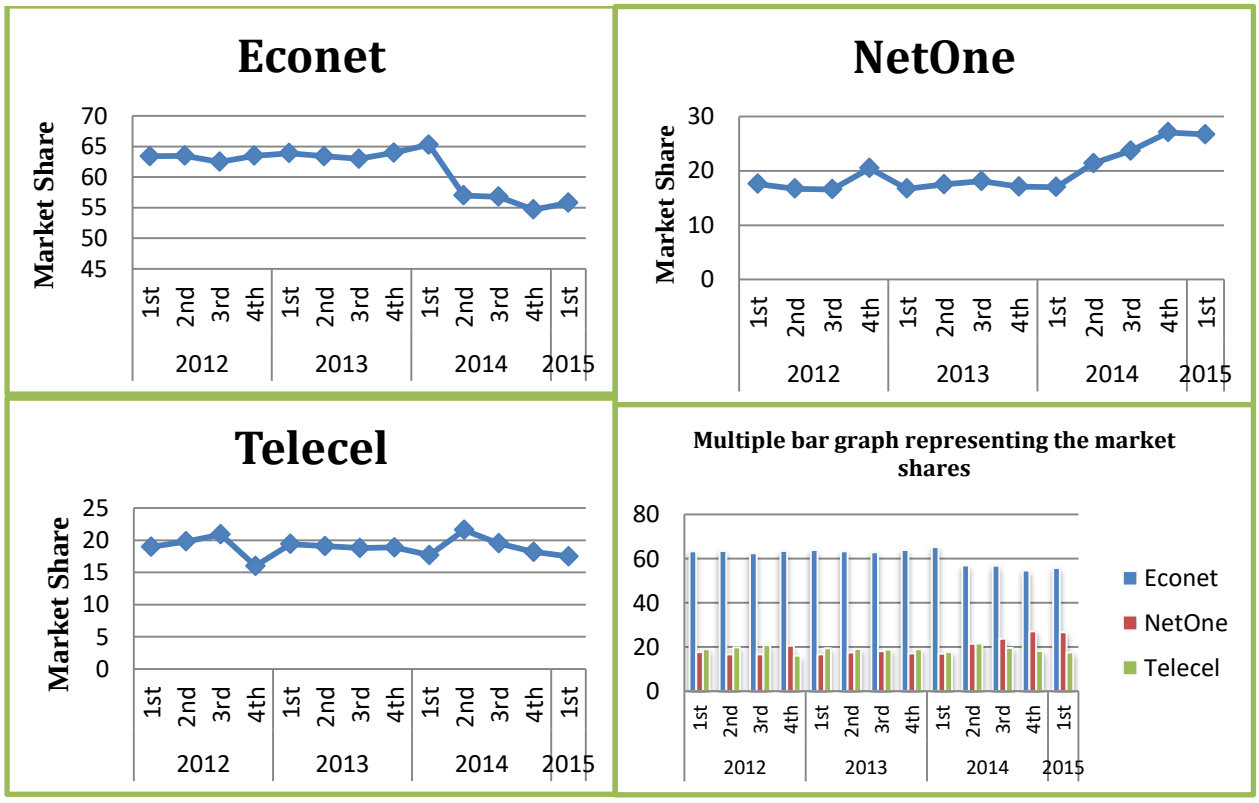
DATA PRESENTATION AND ANALYSIS

4.0 Introduction

This chapter aims at presenting the information on the data collected and the analysis of the interaction of the three mobile service providers in Zimbabwe. It shows the competitive effects of one player to another in terms of subscriber acquisition. Of concern in this chapter are the presentation, analysis, and interpretation of how these three mobile service providers actually affect the others. Thus, graphs and equations shall be used in the illustration and interpretation of the data.

4.1 Graphical representations of the interactions.

Fig 4.1.1



The graphs or trend lines above show the fluctuations in the market shares of the mobile service providers in Zimbabwe from the year 2012 to the 1st quarter of 2015. It can be seen that for the years 2012 and end of 2013 the shares of Econet were stable though fluctuating between 62% and 64%. At the beginning of 2014, Econet experiences a sharp decline in its shares with an increase in the shares of NetOne and Telecel. More so, during the earlier period, the shares did not rise to approach a maximum value and this can be attributed to the existence of competition within the providers.

4.2 Estimated System of Equation and Interpretation of Parameters

The following is the competition model for the three mobile service providers and the parameter results obtained from Genetic Algorithms.

$$\frac{dx_1}{dt} = x_1[0.44 - 0.62x_1 - 0.22x_2 - 0.65x_3]$$

$$\frac{dx_2}{dt} = x_2[0.85 - 0.02x_1 - 1.79x_2 - 0.58x_3] \qquad (7)$$

$$\frac{dx_3}{dt} = x_3[0.23 - 0.06x_1 - 0.14x_2 - 0.51x_3]$$

Where x_1 , x_2 and x_3 are the market shares of Econet, NetOne and Telecel respectively.

The coefficients of the variables provide important information in the analysis of the dynamics of the market shares of the service providers. From the parameter values it can be seen that NetOne and Telecel are obtaining their subscribers from Econet. This is supported by the interspecies competition parameter and this goes hand in glove with the historical data values of the service providers. Econet wireless shares have been increasing since its entry into the market and this can be supported by the high value of its intrinsic growth rate measure parameter.

In equation (2), it can be seen that NetOne has been increasing its subscriber base by taking the subscribers from Econet as shown by the parameter of x_2.

Further more, information pertaining the switching of subscribers from one service provider to the other can also be deduced from the analysis of the parameters. The switching rate for each of the players is shown by the parameter value that corresponds to the interspecies interactions. Thus it can be deduced seen that Econet seems to have been largely affected by its counter competitors NetOne and Telecel. Though the later affected Econet, the earlier tend to have a greater parameter value which shows that Econet was losing its subscribers to NetOne more than to Telecel. This therefore gives important information to the managerial inputs for decision making.

4.3 Critical Points Of The System

The system of equations has got eight critical points or equilibrium solutions. Equilibrium solutions are the values of x_i for which the derivatives of the system of equations equals to zero. Below is a table that shows the critical points of the system:

Table 4.1

CRITICAL POINTS OF THE SYSTEM			
	ECONET	NETONE	TELECEL
1	0.00	0.00	0.00
2	0.00	0.00	0.35
3	0.00	0.48	0.00
4	0.75	0.00	0.00
5	0.00	0.39	0.26
6	0.42	0.00	0.30
7	0.59	0.47	0.00
8	0.56	0.25	0.19

The first equilibrium solution shows the possibility of both players dying out. The practical implication of equilibrium solution 1 is that in the long run, all the telecommunications companies in Zimbabwe will close down. Equilibrium solution 2 represents the possibility of both NetOne and Econet are being competed out by Telecel. The third and the forth can be interpreted in the same manner with the third referring to the survival of NetOne and the forth one referring to the survival of Econet alone. The fifth one refers to the competitive exclusion of Econet and the sixth refers to the competitive exclusion of NetOne. The seventh one refers to the competitive exclusion of Telecel. The last equilibrium solution shows the coexistence of all the telecoms companies.

4.4 Stability Analysis

The system of equations described by (7) is a nonlinear system. Thus the linearization of the system becomes necessary such that one can qualitatively analyse the behaviours of the solutions. For a nonlinear system described by the one below

$$\frac{dX_1}{dx} = P(x_1, x_2, x_3)$$

$$\frac{dX_2}{dx} = Q(x_1, x_2, x_3) \qquad (8)$$

$$\frac{dX_3}{dx} = R(x_1, x_2, x_3)$$

where P, Q and R being functions of $x_1, x_2 \; and \; x_3$ which have continuous partial derivatives up to order 2. Linearization at critical point (X_1^0, X_2^0, X_3^0) is achieved by the following transformation.

$U = X_1 - X_1^0$

$V = X_2 - X_2^0$

$W = X_3 - X_3^0$

Thus the linear system that approximates the nonlinear system (7) near the critical point (X_1^0, X_2^0, X_3^0) is derived using the Jacobian matrix given below.

$$\frac{d}{dt}\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} P_{X_1}(X_1^0, X_2^0, X_3^0) & P_{X_2}(X_1^0, X_2^0, X_3^0) & P_{X_3}(X_1^0, X_2^0, X_3^0) \\ Q_{X_1}(X_1^0, X_2^0, X_3^0) & Q_{X_2}(X_1^0, X_2^0, X_3^0) & Q_{X_3}(X_1^0, X_2^0, X_3^0) \\ R_{X_1}(X_1^0, X_2^0, X_3^0) & R_{X_2}(X_1^0, X_2^0, X_3^0) & R_{X_3}(X_1^0, X_2^0, X_3^0) \end{pmatrix} \begin{pmatrix} U \\ V \\ W \end{pmatrix},$$

where

$P(x_1, x_2, x_3) = x_1[0.44 - 0.62x_1 - 0.22x_2 - 0.65x_3]$

$Q(x_1, x_2, x_3) = x_2[0.85 - 0.02x_1 - 1.79x_2 - 0.58x_3]$

$R(x_1, x_2, x_3) = x_3[0.23 - 0.06x_1 - 0.14x_2 - 0.51x_3]$ (9)

This implies that the Jacobian matrix will be given by:

$$J(X_1X_2,X_3)=\begin{bmatrix} 0.44 - 1.24X_1 - 0.22X_2 - 0.65X_3 & -0.22X_1 & \\ -0.02X_2 & 0.85 - 0.02X_1 - 3.58X_2 - 0.58X_3 & \\ -0.06X_3 & -0.14X_3 & 0.23 - 0.0 \end{bmatrix}$$

For the system of equations described in (7), after the substitution of the critical points or solutions into the Jacobian, the analysis of the Eigen values showed that only one equilibrium solution is stable and the other seven were unstable. For the unstable points, all the trajectories were diverging from the equilibrium solutions as t was increasing.

The general solution for the system in (7) is given by the one below:

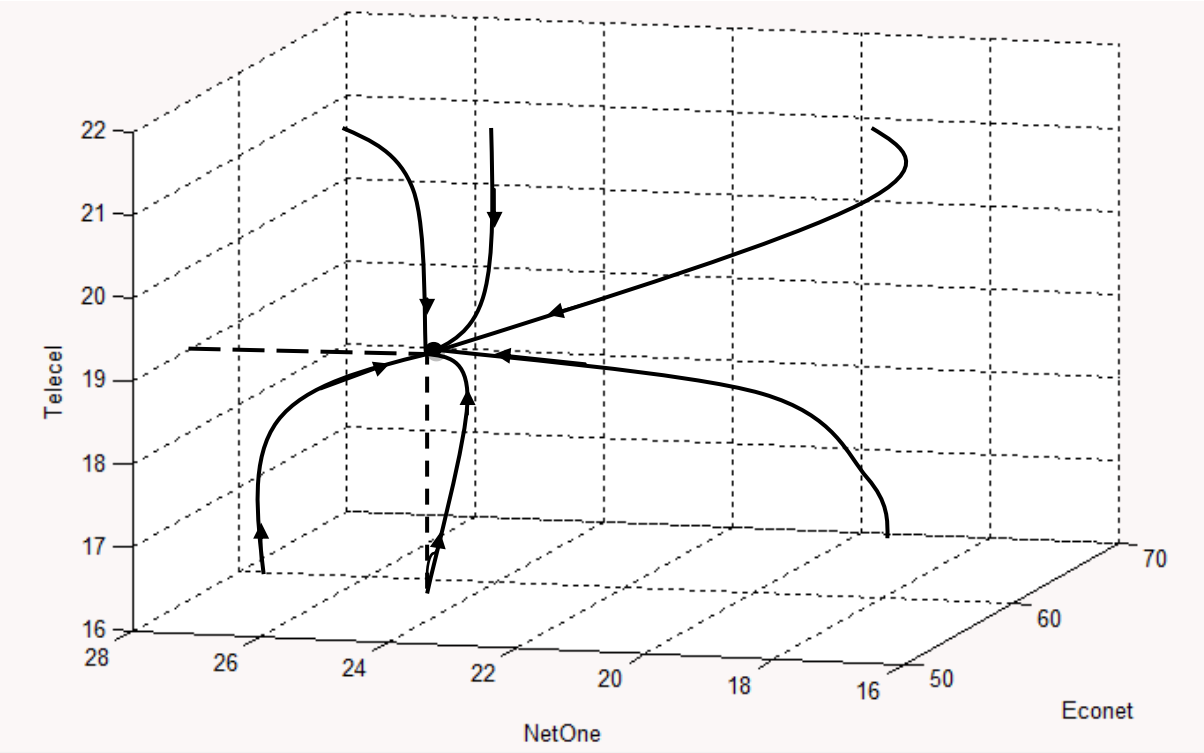
$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = a_1 \begin{pmatrix} -0.24 \\ -0.89 \\ -0.05 \end{pmatrix} e^{-0.69t} + a_2 \begin{pmatrix} -0.88 \\ -0.05 \\ -0.11 \end{pmatrix} e^{-0.32t} + a_3 \begin{pmatrix} 0.86 \\ 0.17 \\ -0.45 \end{pmatrix} e^{-0.17t}$$

where a_1, a_2 and a_3 are constants. Substituting the initial values of the market shares into the general solution gives the values of a_1, a_2 and a_3 and the final solution is given by:

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 0.49 \\ 0.30 \\ 0.24 \end{pmatrix} e^{-0.69t} + \begin{pmatrix} 0.79 \\ 0.16 \\ 0.18 \end{pmatrix} e^{-0.32t} + \begin{pmatrix} -0.321 \\ -0.089 \\ 0.254 \end{pmatrix} e^{-0.17t}$$

From the calculated values, it can be seen that the models were able to accurately describe the dynamics of the markets. This implies that the Lotka-Volterra competition model can be used effectively to analyse the dynamics of market evolution.

Fig 4.2Phase Portraits for Stability Testing



From the phase portrait above, it can be seen that all the trajectories are flowing towards the coexistence equilibrium solution with Econet having a share of 56%, NetOne at 25% and Telecel at 19%.

4.5 Chapter Summary.

The chapter presented data presentation, its analysis and interpretations. It has also presented the research findings and gave an interpretation of them. The next chapter focuses on the recommendations and conclusions of the study

CHAPTER 5

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Summary

The Lotka-Volterra competition model proved to be a useful tool in the estimation of the competitive effects among species that will be interacting in an environment where there is competition for the same limited resources. It is useful in the analysis of relationships that exist when one species in an environment poses a negative effect on its growth rate or that of other coexisting species. This is the case with the Zimbabwean telecommunication companies which are competing for limited resources, which in this case is in the form of the subscribes.

From the fluctuations of the market shares of the service providers, it can be seen that the promotions that are being rolled in by any telecommunication company tend to have a negative effect on the growth rate of the other telecommunications companies.

The analysis however shows that Econet remains the major service provider in Zimbabwe and this can be seen from its shares which are always above the other two service providers, NetOne and Telecel. The growth in Econet's shares may be attributed to a number of promotions which it is running among its clients examples of value added services includes EcoCash,EcoSure and Buddie Zone promotion. However it has been seen that Econet's share have also been dropping at some point and this shows the need for Econet to continue promoting its services otherwise it can lose a number of subscribers to its two rivalry firms as is shown by the down fall in its shares form the 1st quarter of 2014 to the 1st quarter of 2015.

5.2 Conclusions

The main aim of this research was to study the competition among the telecommunications companies in Zimbabwe and to forecast market equilibriums and market concentrations in the Zimbabwean telecommunications sector. The Lotka-Volterra competition model provided a better insight in the analysis of the competitiveness within the sector. The model can be used for making informed decisions by the management of the telecommunications companies and can also provide important information to the telecommunication regulatory authorities with regards to competition in the sector.

5.3 Recommendations

From the analysis of the system, recommendations can be made to NetOne and Telecel that they should increase the resources they use in the promotional strategies such that they can gain and maintain their subscriber bases.

NetOne and Telecel should consider improving their added value services for example One Wallet and Telecash so as to increase their coverage and lure more clients as is done by Econet wireless. As for Econet it should not sit back, it should continue improving its value added services so as to keep on attracting more clients. This is significantly evidenced by a drop in its shares in the 2014 to the 1st quarter of 2015.

With promotions and coverage being the major factors that tend to affect the diffusion of markets, all the service providers should consider improving their coverage and this can now be done easily with the introduction of infrastructure sharing.

5.4 Areas for future research

Future research can include the development of another methodology that could seek to exploit other approaches of the Lotka-Volterra competition model in order to explore the

various aspects that exist in the telecommunications industry. The addition of variables that could represent the promotional strategies that each player is using into the Lotka-Volterra model can also be part of future researches.

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